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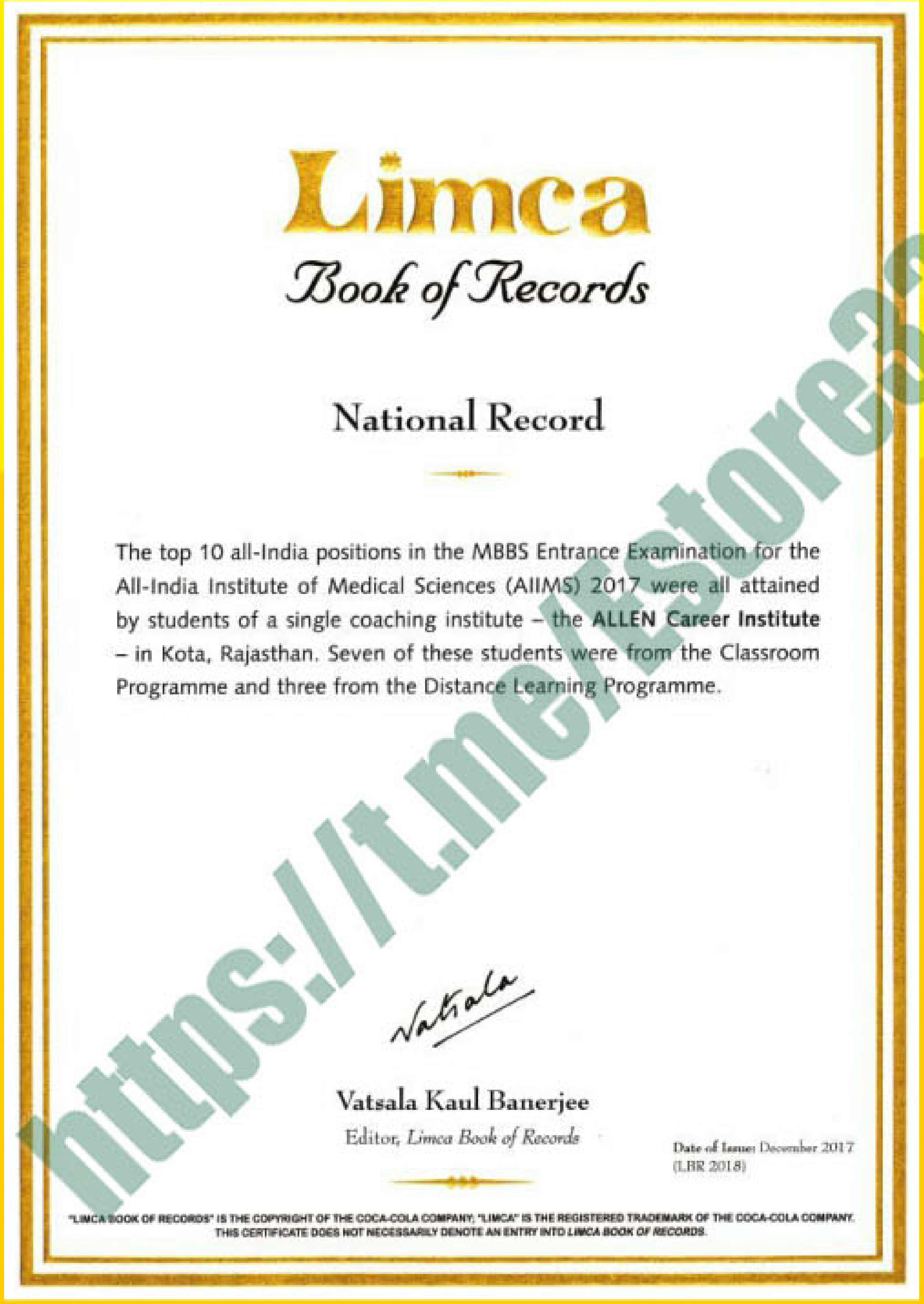

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Vatsala

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Editor, Limca Book of Records

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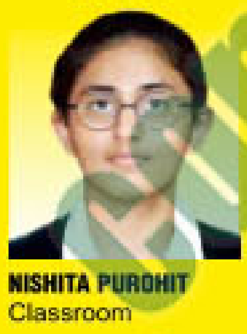
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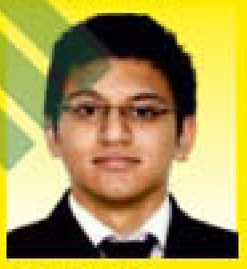
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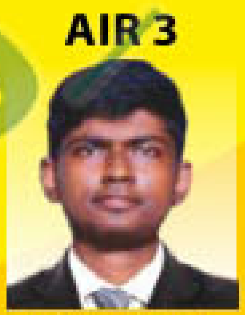
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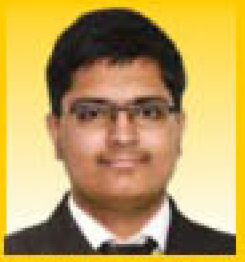
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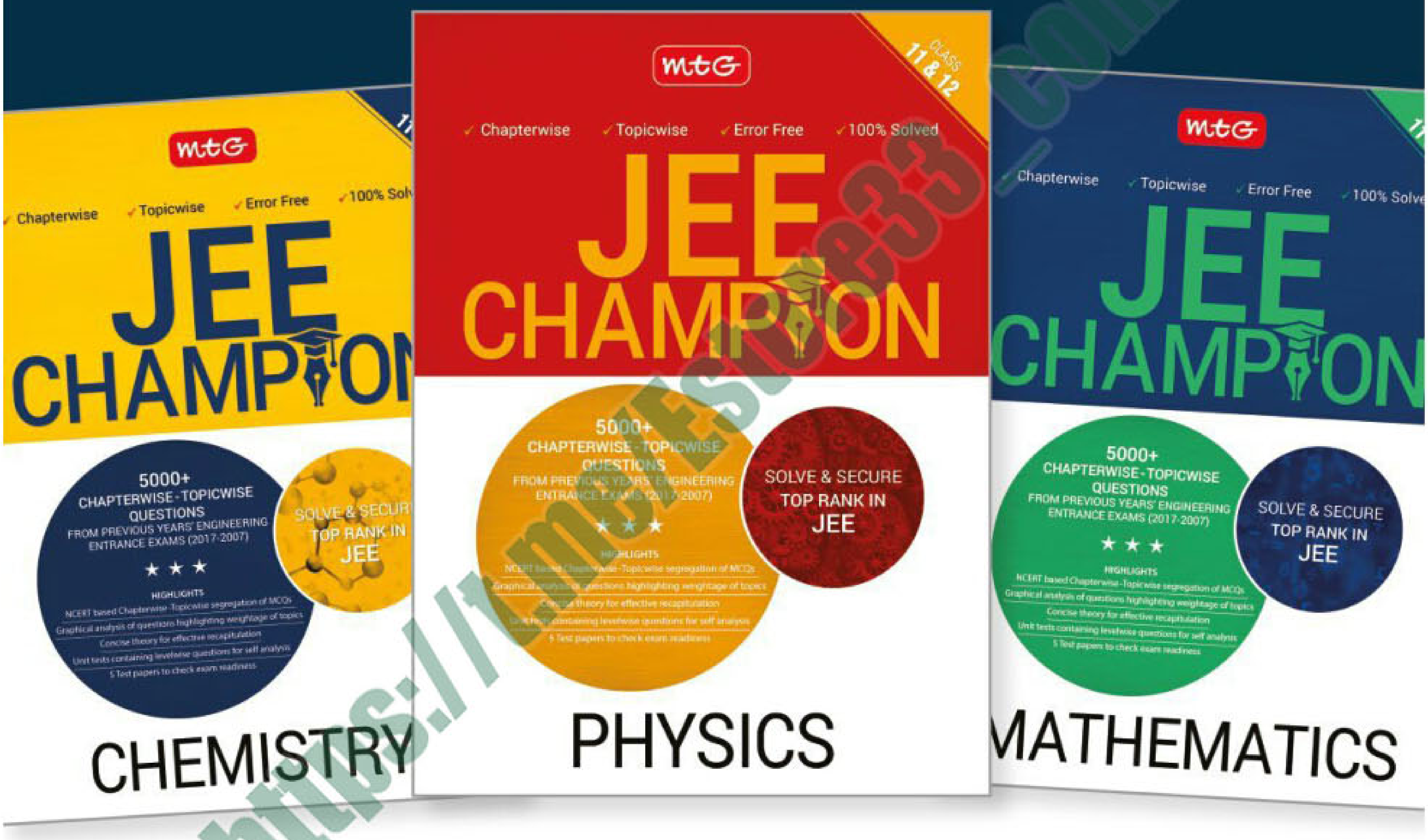
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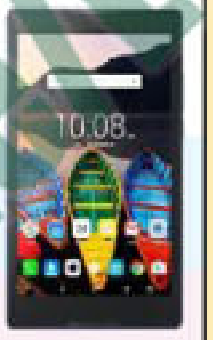

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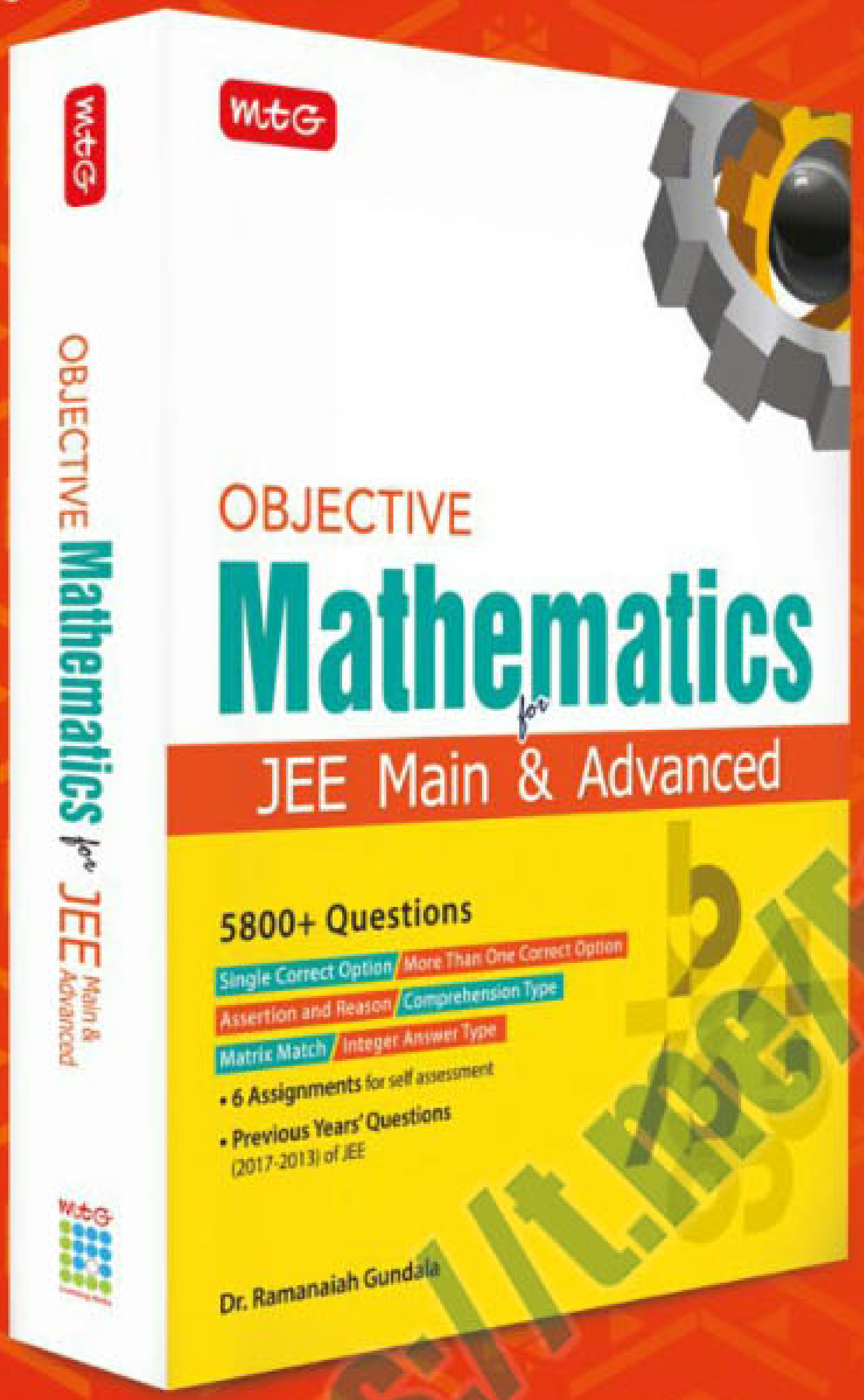
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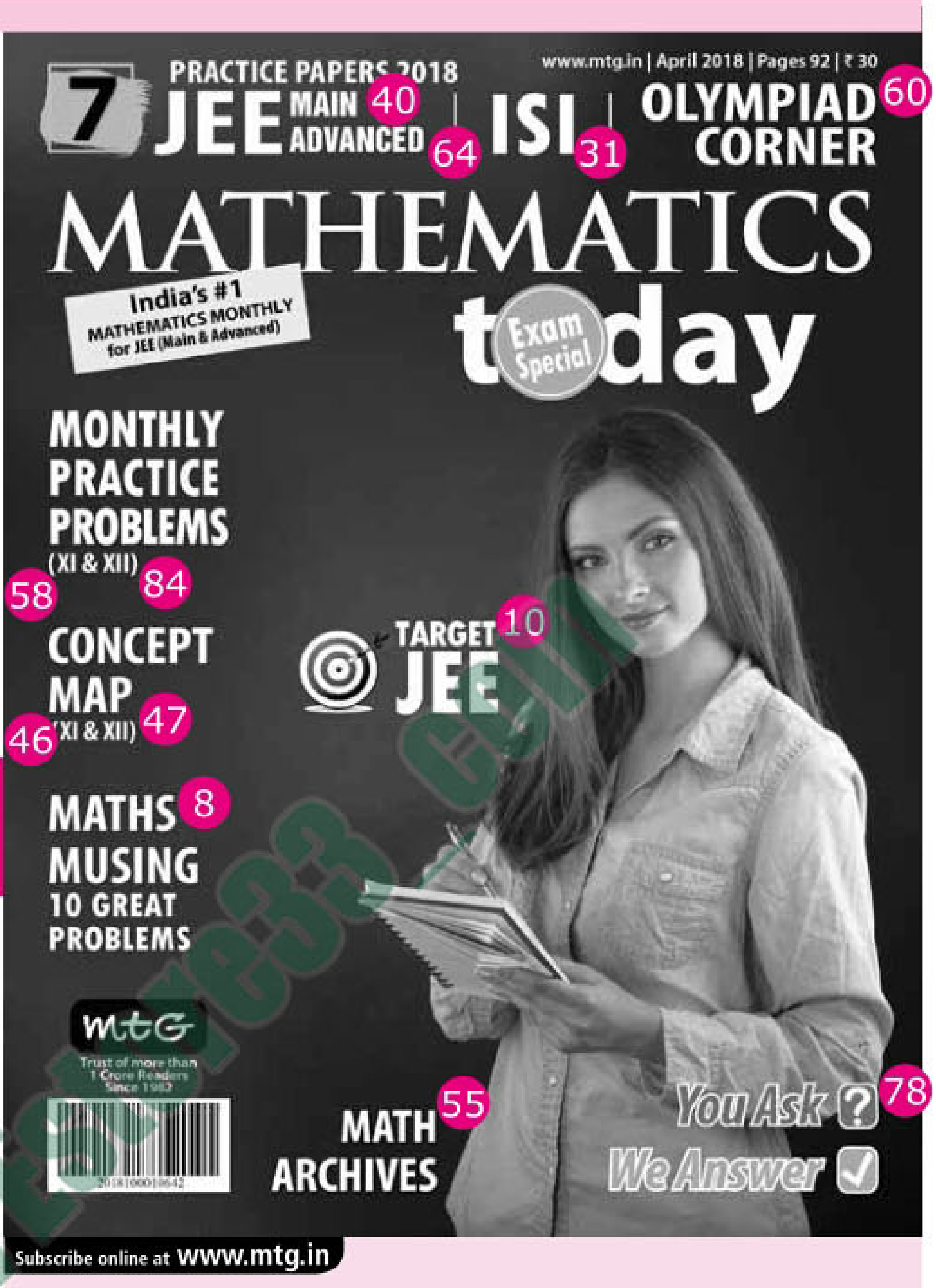
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During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India.

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PROBLEM
Set 184

JEE MAIN

1. If a, b, c are in H.P., b, c, d are in G.P. and c, d, e are in A.P., then $\dfrac{ab^2}{(2a-b)^2}$ is equal to

(a) b (b) a (c) e (d) d

2. How many numbers lying between 100 and 1000 can be formed with the digits 0, 1, 2, 3, 4, 5, if the repetition of the digits is not allowed?

(a) 90 (b) 50 (c) 40 (d) 100

3. If $a = \cos\alpha + i\sin\alpha$ and the equation $az^2 + z + 1 = 0$ has a pure imaginary root, then $\tan\alpha =$

(a) $\dfrac{\sqrt{5}-1}{2}$ (b) $\dfrac{\sqrt{5}+1}{2}$

(c) $\sqrt{\dfrac{\sqrt{5}-1}{2}}$ (d) $\sqrt{\dfrac{\sqrt{5}+1}{2}}$

4. If $\dfrac{d}{dx}F(x) = \dfrac{e^{\sin x}}{x}, x > 0$ and $\displaystyle\int_1^4 \frac{2e^{\sin x^2}}{x}dx = F(k) - F(1)$, then one of the possible values of k is

(a) 4 (b) –4

(c) 16 (d) none of these

5. Let $f(x)$ be a twice differentiable function for all real value of x and satisfies $f(1) = 1, f(2) = 4, f(3) = 9$, then which of the following is definitely true?

(a) $f''(x) = 2\ \forall\ x \in (1, 3)$

(b) $f''(x) = f'(x) = 5$ for some $x \in (2, 3)$

(c) $f''(x) = 3\ \forall\ x \in (2, 3)$

(d) $f''(x) = 2$ for some $x \in (1, 3)$

JEE ADVANCED

6. $f : [0, 1] \to R$ is a differentiable function such that $f(0) = 0$ and $|f'(x)| \le k|f(x)|$ for all $x \in [0, 1], (k > 0)$, then which of the following is/are always true?

(a) $f(x) = 0\ \forall\ x \in R$ (b) $f(x) = 0\ \forall\ x \in [0, 1]$

(c) $f(x) \neq 0\ \forall\ x \in [0, 1]$ (d) $f(1) = k$

COMPREHENSION

For $x \ge 0, f_1(x) = x, f_2(x) = \dfrac{1}{x}, f_3(x) = 1 - x, f_4(x) = \dfrac{1}{1-x}$, $f_5(x) = \dfrac{x-1}{x}, f_6(x) = \dfrac{x}{x-1}$. This family of functions is closed under compositions *i.e.*, composition of any two of these functions is again one of these.

7. Let F be a function such that $f_1\, oF = f_4$. Then F is equal to

(a) f_1 (b) f_2 (c) f_3 (d) f_4

8. Let J be a function such that $f_3 oJof_2 = f_4$. Then J is equal to

(a) f_6 (b) f_5 (c) f_4 (d) f_3

INTEGER TYPE

9. Find the value of $\displaystyle\lim_{x\to\frac{\pi}{2}} \frac{\sin x - (\sin x)^{\sin x}}{1 - \sin x + \ln\sin x}$.

MATRIX MATCH

10. Match the following.

	List-I	List-II	
P.	The distance of the point (3, 0, 5) from the line parallel to the vector $6\hat{i}+\hat{j}-2\hat{k}$ and passing through the point (8, 3, 1) is	1.	12
Q.	If $z^2 + z + 1 = 0$, then $\sum_{r=1}^{6}(z^r + z^{-r})^2$ is	2.	4
R.	If $\int_0^1 \frac{3dx}{1+x^3} = \log K + \frac{\pi}{\sqrt{3}}$ then K equals	3.	3
S.	The number of solutions of the equation $3^{\sin 2x + 2\cos 2x} + 3^{1-\sin 2x + 2\sin 2x} = 28$ in $[0, 2\pi]$ is	4.	2

	P	Q	R	S
(a)	1	2	3	4
(b)	3	1	4	2
(c)	2	3	1	4
(d)	3	2	4	1

See Solution Set of Maths Musing 183 on page no 62

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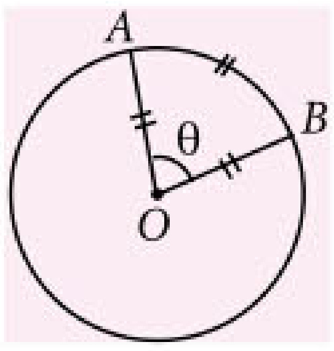
TARGET JEE

Definitions, Formulas and Key Concepts

RADIAN



- Radian is an angle subtended by an arc at the centre of the circle if length of the arc is equal to the radius of the circle.
  i.e. θ = 1 radian if $\overset{\frown}{AB} = OA = r$
  where 'O' is centre of the circle and r be its radius.
  There are three systems of measuring angles namely
  (a) Sexagesimal system (b) French system
  (c) Circular system
  (a) In Sexagesimal system
  1 right angle = 90° (90 degree)
  1° (degree) = 60′ (60 minutes)
  1′ (minute) = 60″ (60 seconds)
  (b) In French system
  1 right angle = 100 grade = 100^g
  1^g (1 grade) = 100 minutes = 100′
  1′ (1 minute) = 100 seconds = 100″
  (c) In Circular systems
  1 radian = $\frac{180}{\pi}$ degree = 57°17′44″ (nearly)
- **Relationship Between Degree, Grade and Radian**
  180° = 200^g = π radian = 2 Right angles
  $$\therefore\ 1^\circ = \left(\frac{10}{9}\right)^g = \left(\frac{\pi}{180}\right)^c$$
  and $1^c = 57^\circ\ 17'\ 44.8'' = 63^g\ 66'\ 20''$

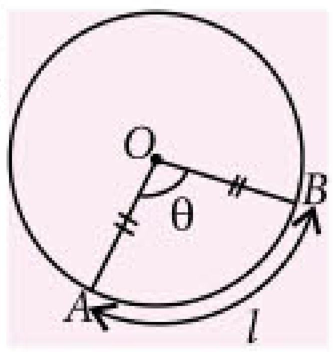
RELATIONSHIP BETWEEN θ, *l* and *r*

Here θ is measure of an angle subtended by an arc of length l and r be the radius of the circle).

We have, $OA = OB = r$

and $\overset{\frown}{AB} = l,\ \angle AOB = \theta$

$$\therefore\ \theta = \frac{l}{r} \quad \text{or} \quad l = r\,\theta$$



DOMAIN AND RANGE OF TRIGONOMETRICAL FUNCTIONS :

Functions	Domain	Range
sinθ	R (set of real numbers)	[–1, 1]
cosθ	R (set of real numbers)	[–1, 1]
tan θ	$R-\left\{(2n+1)\frac{\pi}{2}, n\in I\right\}$	R or (–∞, ∞)
$\cot\theta = \frac{1}{\tan\theta}$	$R - \{n\pi,\ n \in I\}$	R or (–∞, ∞)
$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$	$R - \{n\pi,\ n \in I\}$	R – (–1, 1) or (–∞, –1] ∪ [1, ∞)
$\sec\theta = \frac{1}{\cos\theta}$	$R-\left\{(2n+1)\frac{\pi}{2}, n\in I\right\}$	R – (–1, 1) or (–∞, –1] ∪ [1, ∞)

VALUES OF TRIGONOMETRICAL RATIOS OF SOME ANGLES

- $\sin 15^\circ = \sin(45^\circ - 30^\circ) = \frac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75^\circ$
- $\cos 15^\circ = \cos(45^\circ - 30^\circ) = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75^\circ$
- $\sin 22\frac{1}{2}^\circ = \frac{1}{2}\left(\sqrt{2\sqrt{2}-2}\right)$
- $\cos 22\frac{1^\circ}{2} = \frac{1}{2}\left(\sqrt{\sqrt{2}+2}\right)$
- $\sin 18^\circ = \frac{\sqrt{5}-1}{4} = \cos 72^\circ$
- $\cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4} = \sin 72^\circ$
- $\cos 36^\circ = 2\cos^2 18 - 1 = \frac{\sqrt{5}+1}{4} = \sin 54^\circ$

By : **R. K. Tyagi**, Retd. Principal, HOD Maths, Samarth Shiksha Samiti, New Delhi

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- $\sin 36° = \dfrac{\sqrt{10-2\sqrt{5}}}{4} = \cos 54°$
- $\sin 72°\cos 54° = \sin 36°\cos 18° = \dfrac{\sqrt{5}}{4}$
- $\cos 36°\cos 72° = \sin 54°\sin 18° = \dfrac{1}{4}$
- $\cos 36° - \cos 72° = \sin 54° - \sin 18° = \dfrac{1}{2}$
- $\cot^2 15° = (2+\sqrt{3})^2 = 7+4\sqrt{3} = \tan^2 75°$
- $\tan^2 15° = \cot^2 75° = (2-\sqrt{3})^2 = 7-4\sqrt{3}$
- $\tan 9° = \dfrac{\sin 9°}{\cos 9°} = \dfrac{2\sin^2 9°}{2\sin 9°\cos 9°}$

  $= \dfrac{1-\cos 18°}{\sin 18°} = \dfrac{4-\sqrt{10+2\sqrt{5}}}{\sqrt{5}-1} = \cot 81°$

SUM AND DIFFERENCE FORMULA'S

- $\tan(A+B) = \dfrac{\sin(A+B)}{\cos(A+B)} = \dfrac{\sin A\cos B + \cos A\sin B}{\cos A\cos B - \sin A\sin B}$

  $= \dfrac{\tan A + \tan B}{1-\tan A\tan B}$

  where $A+B \neq (2n\pm 1)\dfrac{\pi}{2}$
- $\tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A\tan B}$ where $A - B$ is not a multiple of $\pi/2$
- $\cot(A+B) = \dfrac{\cot A\cot B - 1}{\cot B + \cot A}$ where $A+B \neq n\pi, n \in I$
- $\sin(A + B)\cdot\sin(A - B) = \sin^2A - \sin^2B$

  $= \cos^2B - \cos^2A$
- $\cos(A + B)\cdot\cos(A - B) = \cos^2A - \sin^2B$

  $= \cos^2B - \sin^2A$
- $\tan(A+B+C) = \dfrac{\sum\tan A - \tan A\tan B\tan C}{1-\sum\tan A\tan B}$
- $\dfrac{\sin(A+B+C)}{\cos A\cos B\cos C} = \sum\tan A - \tan A\tan B\tan C$
- $\dfrac{\cos(A+B+C)}{\cos A\cos B\cos C} = 1-\sum\tan A\tan B$

RESULTS IN PRODUCT FORM

- $(2\cos\theta - 1)(2\cos 2\theta - 1)(2\cos 2^2\theta - 1)\ldots$

  $(2\cos 2^{n-1}\theta - 1) = \dfrac{2\cos 2^n\theta + 1}{2\cos\theta + 1},\ n \in N$
- $\cos\theta\cos 2\theta\cos 2^2\theta\cos 2^3\theta \ldots \cos 2^{n-1}\theta = \dfrac{\sin 2^n\theta}{2^n\sin\theta}$
- $\sin\theta\sin(60° + \theta)\sin(60° - \theta) = \dfrac{1}{4}\sin 3\theta$
- $\cos\theta\cos(60° + \theta)\cos(60° - \theta) = \dfrac{\cos 3\theta}{4}$
- $\tan\theta\tan(60° + \theta)\tan(60° - \theta) = \tan 3\theta$
- $\tan\theta\tan 2\theta\tan 3\theta = \tan 3\theta - \tan 2\theta - \tan\theta$

  (by using $\tan 3\theta = \tan(2\theta + \theta)$)

SUM OF SERIES

- $\sum_{r=0}^{n-1}\sin(\alpha+r\beta) = \sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \ldots + \sin(\alpha + (n-1)\beta)$

  $$= \frac{\sin\left(\alpha+\left(\frac{n-1}{2}\right)\beta\right)\sin\left(\frac{n\beta}{2}\right)}{\sin(\beta/2)} \quad \ldots\text{(i)}$$

  If $\beta = \alpha$ then (i) reduces to

  $\sin\alpha + \sin 2\alpha + \sin 3\alpha + \ldots + \sin(n\alpha)$

  $$= \frac{\sin\left(\frac{n+1}{2}\right)\alpha\,\sin\left(\frac{n\alpha}{2}\right)}{\sin(\alpha/2)}$$
- $\sum_{r=0}^{n-1}\cos(\alpha+r\beta) = \cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \ldots + \cos(\alpha + (n-1)\beta)$

  $$= \frac{\cos\left(\alpha+\frac{(n-1)\beta}{2}\right)\sin\left(\frac{n\beta}{2}\right)}{\sin(\beta/2)} \quad \ldots\text{(ii)}$$

  If $\beta = \alpha$ then (ii) reduces to

  $\cos\alpha + \cos 2\alpha + \cos 3\alpha + \ldots + \cos(n\alpha)$

  $$= \frac{\cos\left(\frac{n+1}{2}\right)\alpha\cdot\sin\left(\frac{n\alpha}{2}\right)}{\sin(\alpha/2)}$$

FORMULAS FOR sin 2θ, cos2θ, tan2θ, sin3θ, cos3θ, tan3θ and sin*n*θ, cos*n*θ, tan*n*θ

- $\sin 2\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$
- $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ and $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$
- $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$
- $\tan 4\theta = \dfrac{4\tan\theta - 4\tan^3\theta}{1-6\tan^2\theta+\tan^4\theta}$

  $= \dfrac{4\tan\theta(1-\tan^2\theta)}{1-6\tan^2\theta+\tan^4\theta}$

MAXIMUM AND MINIMUM VALUES OF TRIGONOMETRICAL FUNCTIONS

- Let $y = a\sin nx + b\cos nx$ then

  $-\sqrt{a^2+b^2} \leq y \leq \sqrt{a^2+b^2}$
- $\dfrac{3}{4} \leq \sin^2 x + \cos^4 x \leq 1$

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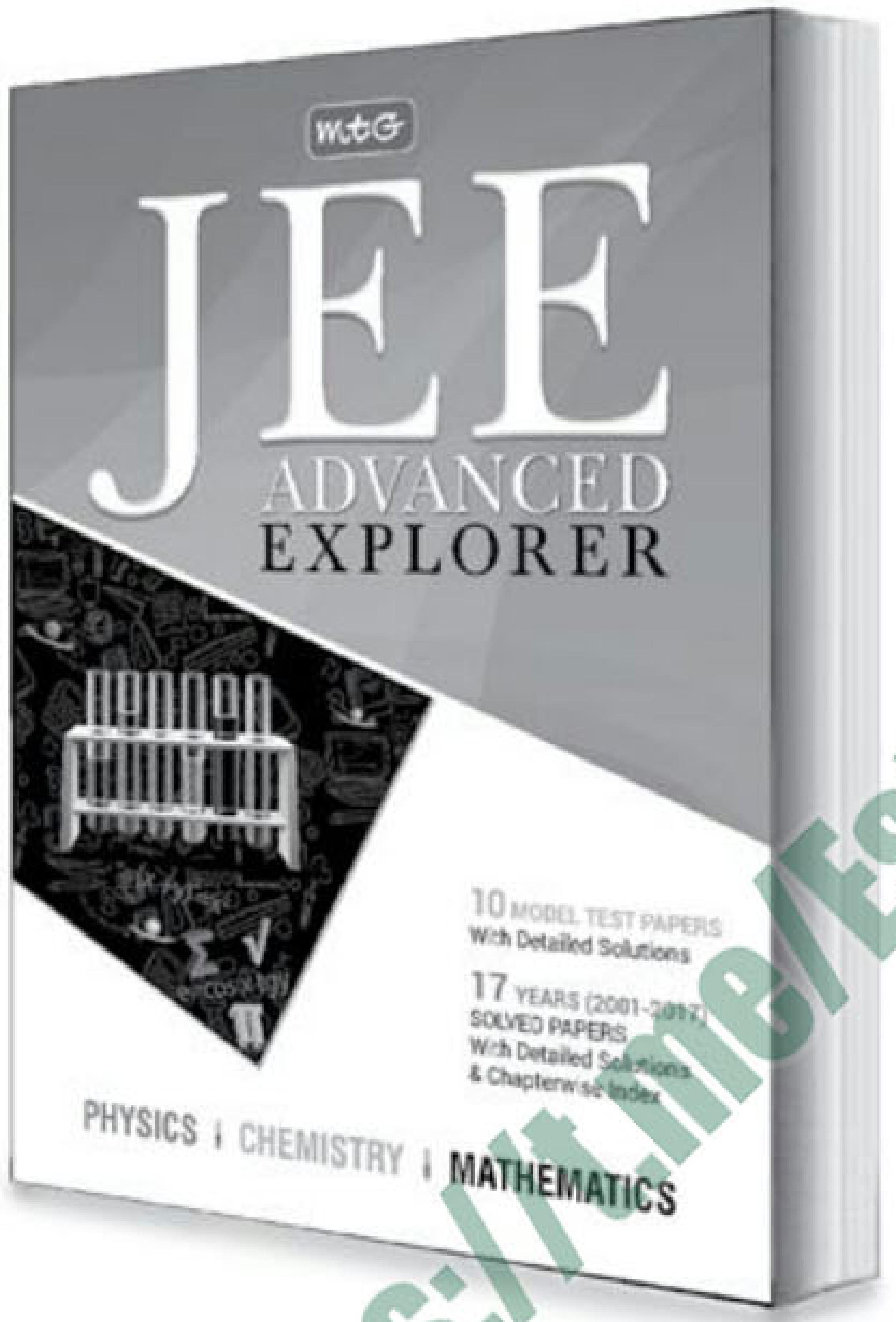
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or $\frac{3}{4} \le \sin^2 x + \cos^4 x = \cos^2 x + \sin^4 x \le 1 \; \forall \; x$

- $-\frac{1}{4} \le \sin\theta \; \sin(60° + \theta)\sin(60° - \theta) \le \frac{1}{4}$ and

  $-\frac{1}{4} \le \cos\theta \; \cos(60° + \theta)\cos(60° - \theta) \le \frac{1}{4}$ and

  $-\frac{1}{4} \le \cos\theta \; \cos(120° - \theta)\cos(120° + \theta) \le \frac{1}{4}$
- $-1 \le \frac{3\sin x - 4\sin^3 x}{4} \le 1$
- $\frac{\sin(\alpha+\beta+\gamma)}{\sin\alpha + \sin\beta + \sin\gamma} \le 1 \; \forall \; \alpha, \beta, \gamma \in \left(0, \frac{\pi}{2}\right)$
- $(\sin\theta + \text{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 \ge 9$
- If $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ then $a\sec\theta + b\cos\theta \ge 2\sqrt{ab}$, by using the concept AM ≥ GM and there will be no change in least value if a and b are interchanged.
- If $0 < \theta < \pi$ then $a\text{cosec}\theta + b\sin\theta \ge 2\sqrt{ab}$
- $a^{\cos^2 x} + a^{\sin^2 x} \ge 2\sqrt{a^{\cos^2 x + \sin^2 x}} = 2\sqrt{a}$

  (Using AM ≥ GM)
- $a^2\sec^2\theta + b^2\text{cosec}^2\theta \le 2ab$
- If $\theta \in \left(0, \frac{\pi}{2}\right)$ and $a, b > 0$ then $f(\theta) = a\sec\theta + b\,\text{cosec}\theta$ will be minimum at $\theta = \tan^{-1}\left(\sqrt[3]{\frac{b}{a}}\right)$ and its mininum value is $\left(a^{\frac{2}{3}} + b^{\frac{2}{3}}\right)^2$. It means $f(\theta)$ will attains minimum value when $\tan^3\theta = b/a$.
- **In a triangle *ABC***

  The expressions sin*A* + sin*B* + sin*C* and sin*A* sin*B* sin*C* attains their maximum values when *A* = *B* = *C* = 60° (thus triangle is equilateral)

  ∴ max(sin*A* + sin*B* + sin*C*) = $\frac{3\sqrt{3}}{2}$

  and max(sin*A* sin*B* sin*C*) = $\frac{3\sqrt{3}}{8}$. Also

  (1) $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} \le \frac{1}{8}$

  (2) $\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2} \le \frac{3\sqrt{3}}{8}$

  (3) $\cos A + \cos B + \cos C \le \frac{3}{2}$

  (4) $\tan A + \tan B + \tan C \ge 3\sqrt{3}$

  and $\tan A \tan B \tan C \ge 3\sqrt{3}$

  (attains least value at *A* = *B* = *C* = 60°)

  $\cot A \cot B \cot C \le \frac{1}{3\sqrt{3}}$

  (attains max. value at *A* = *B* = *C* = 60°)

  (5) $\tan^2\frac{A}{2} + \tan^2\frac{B}{2} + \tan^2\frac{C}{2} \ge 1$

  (6) cos*A* cos*B* cos*C* ≤ 1/8
- $\frac{\sin^2 A + \sin A + 1}{\sin A} \ge 3$, $\sum\left(\frac{\sin^2 A + \sin A + 1}{\sin A}\right) \ge 9$,

  $\prod\left[\frac{\sin^2 A + \sin A + 1}{\sin A}\right] \ge 27$
- If *A* be the area of triangle and *s* be semi perimeter of Δ then

  $A \le \frac{s^2}{3\sqrt{3}}$ or $3\sqrt{3}A \le s^2$
- $(s-a)(s-b)(s-c) \le \frac{abc}{8}$

  $\frac{1}{s-a} + \frac{1}{s-b} + \frac{1}{s-c} \ge \frac{2(ab+bc+ca)}{abc}$
- (a) If $y = \frac{1+\tan^2\theta - \tan\theta}{1+\tan^2\theta + \tan\theta}$ then $\frac{1}{3} \le y \le 3$
- (b) If $y = \frac{\tan\theta}{\tan 3\theta}$ then $y < \frac{1}{3}$ or $y > 3$
- If *R* denotes the circumradius of a triangle then

  a cot*A* + *b* cot*B* + *c* cot*C* ≤ 3*R*
- If *a*, *b*, *c* are sides opposite to the angles *A*, *B*, *C* respectively then

  $bc(b+c)\cos A + ca(c+a)\cos B + ab(a+b)\cos C \ge 3abc$

CONDITIONAL TRIGONOMETRICAL IDENTITIES

- If *A* + *B* + *C* = π, then

  sin2*A* + sin2*B* + sin2*C* = 4sin*A* sin*B* sin*C* and

  sin*A* + sin*B* + sin*C* = $4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$
- sin*A* + sin*B* – sin*C* = $4\sin\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$, similarly others.
- cos2*A* + cos2*B* + cos2*C* = –1 – 4cos*A* cos*B* cos*C*

  and cos*A* + cos*B* + cos*C* = $1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$
- cos2*A* + cos2*B* – cos2*C* = –1 – 4sin*A* sin*B* sin*C*
- cos*A* + cos*B* – cos*C* = $-1 + 4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

- $\dfrac{\cos A}{\sin B\sin C}+\dfrac{\cos B}{\sin C\sin A}+\dfrac{\cos C}{\sin A\sin B}=2$
- $\sin^2A+\sin^2B-\sin^2C=2\sin A\sin B\cos C$
- $\cos^2A+\cos^2B+\cos^2C=1-2\cos A\cos B\cos C$
- $\tan A+\tan B+\tan C=\tan A\tan B\tan C$
- $\cot A\cot B+\cot B\cot C+\cot C\cot A=1$
- $\tan\dfrac{A}{2}\tan\dfrac{B}{2}+\tan\dfrac{B}{2}\tan\dfrac{C}{2}+\tan\dfrac{C}{2}\tan\dfrac{A}{2}=1$
- $\cot\dfrac{A}{2}+\cot\dfrac{B}{2}+\cot\dfrac{C}{2}=\cot\dfrac{A}{2}\cot\dfrac{B}{2}\cot\dfrac{C}{2}$
- $\cos\dfrac{A}{2}+\cos\dfrac{B}{2}+\cos\dfrac{C}{2}$
$$=4\cos\frac{A+B}{4}\cos\frac{B+C}{4}\cos\frac{C+A}{4}$$
$$=4\cos\left(\frac{\pi-A}{4}\right)\cos\left(\frac{\pi-B}{4}\right)\cos\left(\frac{\pi-C}{4}\right)$$
- $\dfrac{\sin 2A+\sin 2B+\sin 2C}{\sin A+\sin B+\sin C}=8\sin\dfrac{A}{2}\sin\dfrac{B}{2}\sin\dfrac{C}{2}$
- $\dfrac{\sin 2A+\sin 2B+\sin 2C}{\cos A+\cos B+\cos C-1}=8\cos\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2}$

If $A+B+C=\pi/2$

- $\sin 2A+\sin 2B+\sin 2C=4\cos A\cos B\cos C$
- $\sin^2A+\sin^2B+\sin^2C=1-2\sin A\sin B\sin C$

PERIODIC FUNCTIONS

- A function $f(x)$ is said to be periodic function if $f(x+p)=f(x)$, where least positive value of p is known as period of $f(x)$.
  (1) If $f(x)$ is periodic with period p, then period of $k\,f\,[c(x+p)]$ is $\dfrac{p}{|c|}$.
  (2) Let $f_1(x), f_2(x), \ldots, f_n(x)$ are periodic functions with periods $p_1, p_2, p_3, \ldots, p_n$ then the period of $h(x)=a_1f_1(x)\pm a_2f_2(x)\pm a_3f_3(x)\pm \ldots \pm a_nf_n(x)$ is equal to the L.C.M of $(p_1, p_2, p_3, \ldots, p_n)$ provided $h(x)$ is not an even function.
  (3) (a) The L.C.M of a rational is possible with other rational.
    (b) L.C.M of an irrational with other irrational is possible.
    (c) L.C.M of an irrational and a rational is not possible. For example L.C.M. of $(2\pi, \pi, 1)$ is not possible as L.C.M. of $(\pi, 2\pi)$ is 2π.

SOLUTION OF TRIGONOMETRIC EQUATIONS

A value of unknown angle which satisfies the given equation is called the solution of the equation.

- Type of Solutions

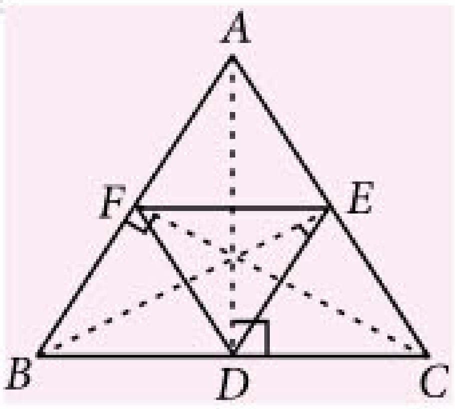
(1) **Principal solution :** The smallest numerical value either positive or negative for the angle of a trigonometrical equation is called principal solution. Suppose there exist smallest positive and smallest negative values but equal then we count the positive value as the principal values.

(2) **General solution of trigonometric equations**
  (a) $\sin x=0$ is $x=\pi k,\ k\in I$
  (b) $\cos x=0 \Rightarrow x=(2k\pm1)\dfrac{\pi}{2}$
  (c) $\tan x=0 \Rightarrow x=k\pi,\ k\in I$
  (d) (i) $\sin x=\sin\alpha \Rightarrow x=(-1)^k\alpha+\pi k,\ k\in I$
    (ii) $\cos x=\cos\alpha \Rightarrow x=2k\pi\pm\alpha,\ k\in I$
    (iii) $\tan x=\tan\alpha \Rightarrow x=k\pi+\alpha,\ k\in I$
  (e) $\sin^2x=\sin^2\alpha$, $\cos^2x=\cos^2\alpha$, $\tan^2x=\tan^2\alpha$ each having identical (same) solution given by $x=k\pi\pm\alpha,\ k\in I$ and α is called principal angle.

CONCEPT OF PEDAL TRIANGLE

Let ABC be any triangle and AD, BE, CF are perpendicular's drawn from A, B, C to the opposite sides BC, CA, AB respectively. The point of concurrency of these perpendiculars (altitudes) of the triangle ABC is called orthocentre of the triangle

The triangle formed by joining the feet of these perpendiculars is called pedal triangle. In the figure, ΔDEF is the pedal triangle of ΔABC.

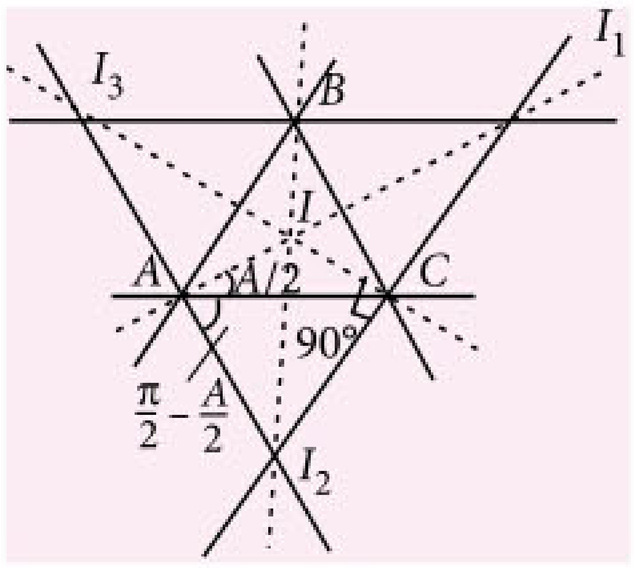


∴ Sides of pedal triangle are

- $DE=c\cos C=2R\sin C\cos C=R\sin 2C$
- $EF=a\cos A=2R\sin A\cos A=R\sin 2A$
- $DF=b\cos B=2R\sin B\cos B=R\sin 2B$
- Inradius $=2R\cos A\cos B\cos C$, Circumradius $=R/2$
- Area of pedal triangle $=2\Delta\cos A\cos B\cos C$
$$=\frac{1}{2}R^2\sin 2A\sin 2B\sin 2C$$

EXCENTRAL TRIANGLE

The triangle formed by joining the three excentres I_1, I_2 and I_3 of ΔABC is called excentral or excentric triangle and we have the following results.

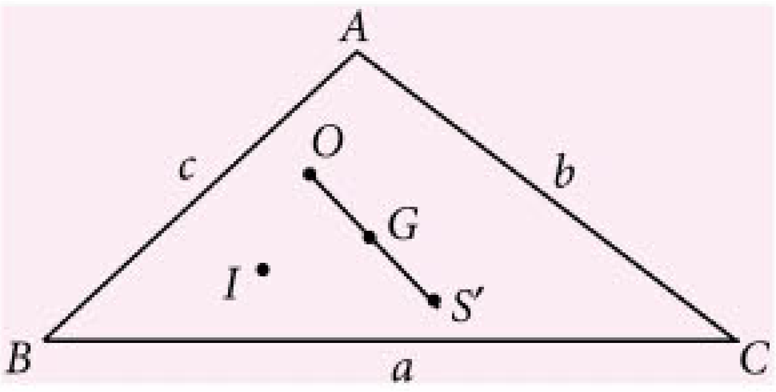


- Incentre I of the ΔABC is the orthocentre of the excentral $\Delta I_1I_2I_3$.
- ΔABC is the pedal triangle of the $\Delta I_1I_2I_3$.

- The angles of excentral triangle are
  $\frac{\pi}{2}-\frac{A}{2}, \frac{\pi}{2}-\frac{B}{2}, \frac{\pi}{2}-\frac{C}{2}$
- The sides of excentral triangle are
  $4R\cos\frac{A}{2},\ 4R\cos\frac{B}{2},\ 4R\cos\frac{C}{2}$
- $II_1 = 4R\sin\frac{A}{2},\ II_2 = 4R\sin\frac{B}{2},\ II_3 = 4R\sin\frac{C}{2}$

BASIC PROPERTIES OF TRIANGLES WITH NOTATIONS

Let us consider a triangle *ABC*.



Notations : *A*, *B*, *C* are angles and *a*, *b*, *c* are sides opposite to angles *A*, *B*, *C* respectively.

- *G* is centroid, which is point of intersection of medians.
- *O* is orthocentre, which is point of intersection of perpendiculars drawn from vertices to opposite sides.
- *S*′ is circumcentre which is point of concurrency of perpendicular bisectors of the triangle.
- *I* is the incentre, the point of concurrency of internal bisectors of the angles of the triangle.
- I_1, I_2, I_3 are excentres, which are point of intersection of internal bisector of one angle and external bisectors of other two angles.
- *r* is inradius, the radius of the circle with centre *I* which touches the sides of the triangle.
- *R* is circumradius, which is the radius of circle with centre *S* which passes through the vertices *A*, *B*, *C*.
- r_1, r_2, r_3 are ex-radii, the radii of circles with centres I_1, I_2, I_3 respectively which touches the sides of the triangle.
- *s* is the semiperimeter which equals $\frac{a+b+c}{2}$ or $2s = a + b + c$ = perimeter of triangle.

RADIUS OF EX-CIRCLES

- Let r_1, r_2, r_3 are ex-radius and *a*, *b*, *c* are sides of the triangle ABC, then

$$r_1 = \frac{\Delta}{s-a} = \sqrt{\frac{s(s-b)(s-c)}{s-a}},$$

$$r_2 = \frac{\Delta}{s-b} = \sqrt{\frac{s(s-a)(s-c)}{s-b}},$$

$$r_3 = \frac{\Delta}{s-c} = \sqrt{\frac{s(s-a)(s-b)}{s-c}}$$

Finally, we say

(1) $r_1 = \frac{\Delta}{s-a} = s\tan\frac{A}{2} = \frac{a\cos\frac{B}{2}\cos\frac{C}{2}}{\cos\frac{A}{2}}$

$$= 4R\cos\frac{B}{2}\cos\frac{C}{2}\sin\frac{A}{2}$$

(2) $r_2 = \frac{\Delta}{s-b} = s\tan\frac{B}{2} = \frac{b\cos\frac{A}{2}\cos\frac{C}{2}}{\cos\frac{B}{2}}$

$$= 4R\cos\frac{A}{2}\cos\frac{C}{2}\sin\frac{B}{2}$$

(3) $r_3 = \frac{\Delta}{s-c} = s\tan\frac{C}{2} = \frac{c\cos\frac{A}{2}\cos\frac{B}{2}}{\cos\frac{C}{2}}$

$$= 4R\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$$

(4) $\frac{1}{r} = \frac{s}{\Delta} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$

(5) $r_1 + r_2 + r_3 = 4R + r$, *r* is inradius of ΔABC.

(6) $r.r_1r_2r_3 = \Delta^2$

- $(r_1 - r)(r_2 - r)(r_3 - r) = 4Rr^2$
- $\left(\frac{1}{r}-\frac{1}{r_1}\right)\left(\frac{1}{r}-\frac{1}{r_2}\right)\left(\frac{1}{r}-\frac{1}{r_3}\right) = \frac{16R}{r^2(a+b+c)^2}$

$$= \frac{4R}{(rs)^2} = \frac{4R}{(\Delta)^2} = \frac{abc}{\Delta^3}$$

- $r^2 + r_1^2 + r_2^2 + r_3^2 = 16R^2 - (a^2 + b^2 + c^2)$
- $a\cos A + b\cos B + c\cos C = \frac{r(a+b+c)}{R}$

$$= \frac{r}{R}(\text{perimeter of triangle})$$

or $\frac{R(a\cos A + b\cos B + c\cos C)}{a+b+c} = r$

or $\frac{\sin 2A + \sin 2B + \sin 2C}{\sin A + \sin B + \sin C} = \frac{2r}{R}$

- $r_1\, r_2\, r_3 = r^3\left(\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}\right)^2$

- $$\frac{1}{r^2}+\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}=\frac{a^2+b^2+c^2}{\Delta^2}$$
  (Δ = area of triangle = rs)
- $$a\cot A+b\cot B+c\cot C=\frac{a}{\tan A}+\frac{b}{\tan B}+\frac{c}{\tan C}$$
  $$=\frac{a\cos A}{\sin A}+\frac{b\cos B}{\sin B}+\frac{c\cos C}{\sin C}=2(R+r)$$
- $r + r_1 + r_2 + r_3 = 4R\cos C$ and so on.
- **Cyclic results**

(1) $$\frac{\sin B-\sin C}{r_1}+\frac{\sin C-\sin A}{r_2}+\frac{\sin A-\sin B}{r_3}=0$$
$$=\frac{b-c}{r_1}+\frac{c-a}{r_2}+\frac{a-b}{r_3}$$

(2) $$(r+r_1)\tan\left(\frac{B-C}{2}\right)+(r+r_2)\tan\left(\frac{C-A}{2}\right)$$
$$+\ (r+r_3)\tan\left(\frac{A-B}{2}\right)=0$$

(3) $$\frac{r_1+r_2}{1+\cos C}=\frac{r_3+r_1}{1+\cos B}=\frac{r_2+r_3}{1+\cos A}$$

(4) $2Rc\sin(A - B) + 2Rb(\sin C - A)$
$+ 2Ra\sin(B - C) = 0$

($\because$ $a\sin(B - C) = \sin^2 B - \sin^2 C$ and so on).

- If p_1, p_2, p_3 are perpendiculars (length) drawn from the vertices to opposite side of the triangle, then

(1) $$\frac{1}{p_1}+\frac{1}{p_2}+\frac{1}{p_3}=\frac{1}{r} \text{ and } p_1p_2p_3=\frac{(abc)^2}{8R^2}$$

(2) $$\frac{\cos A}{p_1}+\frac{\cos B}{p_2}+\frac{\cos C}{p_3}=\frac{1}{R}$$

(3) $$\frac{b}{c}p_1+\frac{c}{a}p_2+\frac{a}{b}p_3=\frac{a^2+b^2+c^2}{2R}$$

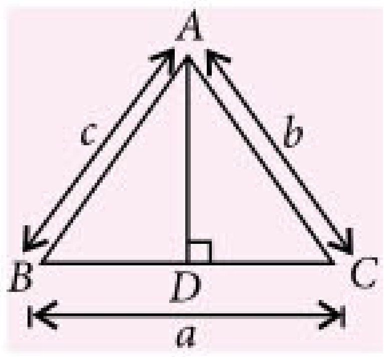
(4) $$\frac{1}{p_1}+\frac{1}{p_2}-\frac{1}{p_3}=\frac{s-c}{\Delta}=\frac{2ab}{(a+b+c)\Delta}\cos^2\frac{C}{2}$$
and similarly others.

(5) $$\frac{1}{p_1^2}+\frac{1}{p_2^2}+\frac{1}{p_3^2}=\frac{a^2+b^2+c^2}{4\Delta^2}$$
$$=\frac{\cot A+\cot B+\cot C}{\Delta}$$

PROPERTIES OF TRIANGLES

- **Sine Formula**

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$



- **Cosine Formula**

(1) $\cos A=\dfrac{b^2+c^2-a^2}{2bc}$ (2) $\cos B=\dfrac{c^2+a^2-b^2}{2ac}$

(3) $\cos C=\dfrac{a^2+b^2-c^2}{2ab}$

- **Projection Formula**

(1) $a = c\cos B + b\cos C$ (2) $b = a\cos C + c\cos A$
(3) $c = a\cos B + b\cos A$

- **Half Angle Formula**

- $\sin\dfrac{A}{2}=\sqrt{\dfrac{(s-b)(s-c)}{bc}}$ • $\cos\dfrac{A}{2}=\sqrt{\dfrac{s(s-a)}{bc}}$

- **Tangent Rule (Napier's Analogy)**

(i) $$\tan\left(\frac{A-B}{2}\right)=\frac{a-b}{a+b}\cot\frac{C}{2}$$

(ii) $$\tan\left(\frac{B-C}{2}\right)=\frac{b-c}{b+c}\cot\frac{A}{2}$$

(iiii) $$\tan\left(\frac{C-A}{2}\right)=\frac{c-a}{c+a}\cot\frac{B}{2}$$

AREA OF Δ IN DIFFERENT FORMS

Let Δ denotes the area of a triangle then

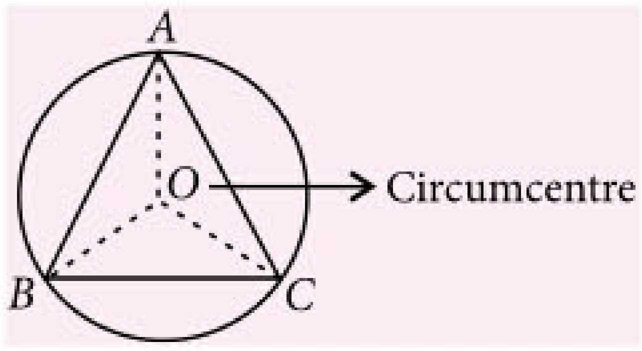
- Hero's formula $\Delta=\sqrt{s(s-a)(s-b)(s-c)}$ where $2s = a + b + c$ where a, b, c are side of the triangle opposite to the vertices A, B, C respectively.
  $$=\frac{1}{4}\sqrt{(a+b+c)(a+b-c)(b+c-a)(c+a-b)}$$
- $$\Delta=\frac{1}{2}ab\sin C=\frac{1}{2}ac\sin B=\frac{1}{2}bc\sin A$$
- $$\Delta=\frac{c^2\sin A\sin B}{2\sin C}=\frac{b^2\sin A\sin C}{2\sin B}=\frac{a^2\sin B\sin C}{2\sin A}$$
- $$\Delta=\frac{1}{2}\text{base}\times\text{height}$$
- For equilateral triangle, $\Delta=\dfrac{\sqrt{3}}{4}(\text{side})^2$

Note : The above formula are applicable in all type of triangles *i.e.*, in equilateral, isosceles, scalene and right angle, obtuse angle as well as acute angle triangle.

CIRCUMCIRCLE AND CIRCUMRADIUS

Circumcircle of a triangle and circumradius (R)

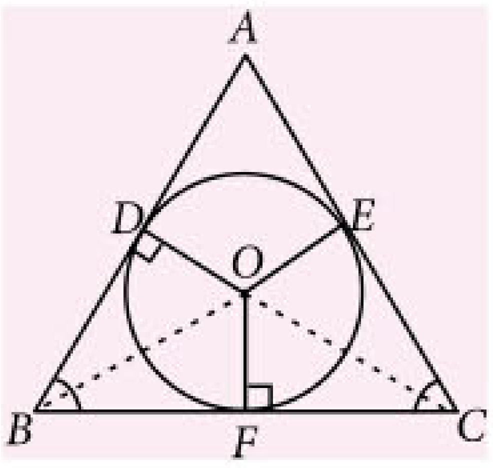
$OA = OB = OC = R$, where O is point of intersection of perpendicular bisectors of sides of the triangle ABC



- $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R$
- $R=\frac{abc}{4\Delta}=\frac{abc}{4\sqrt{s(s-a)(s-b)(s-c)}}$

INSCRIBED CIRCLE AND INRADIUS

Here, 'O' is the point of intersection of internal angle bisectors known as incentre where $OE = OF = OD$ = length of ⊥ drawn from incentre to any one of the side of the triangle



- $r=\frac{\Delta}{s}$ (r = in radius of ΔABC) *i.e.*, $\Delta = rs$
- $r\cot\frac{A}{2}=s-a, r\cot\frac{B}{2}=s-b, r\cot\frac{C}{2}=s-c$

  On adding these, we get

  $$r\left(\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}\right)=3s-2s=s$$

  $$\Rightarrow \cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}=\frac{s}{r}$$

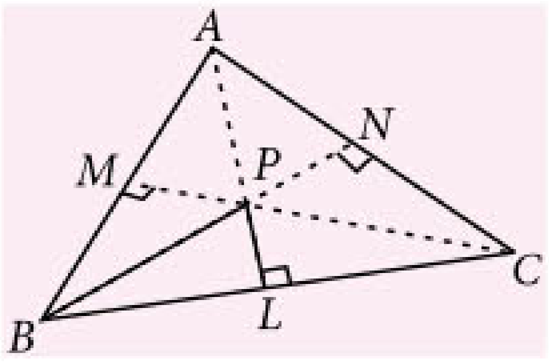
  $$\Rightarrow \frac{\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}}{a+b+c}=\frac{1}{2r}$$

  or $$\frac{a+b+c}{2\left(\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}\right)}=r$$

- $r=\frac{a\sin\frac{B}{2}\sin\frac{C}{2}}{\cos\frac{A}{2}}, r=\frac{b\sin\frac{A}{2}\sin\frac{C}{2}}{\cos\frac{B}{2}}, r=\frac{c\sin\frac{A}{2}\sin\frac{B}{2}}{\cos\frac{C}{2}}$
- $r=4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

ORTHOCENTRE

The point of intersection of ⊥'s drawn from vertices to opposite sides is called orthocentre.

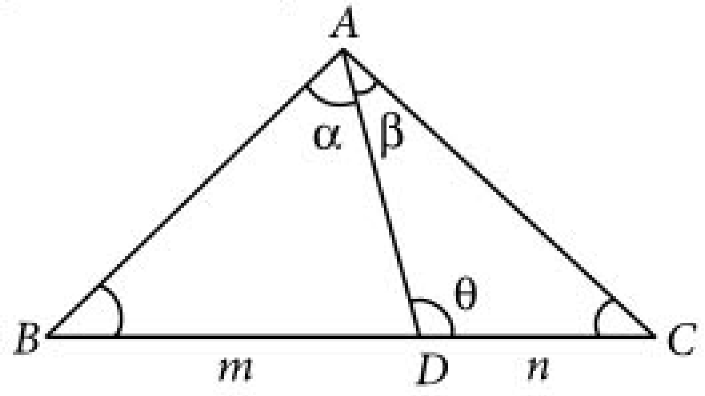


- Distance of orthocentre from the vertices is PA, PB, PC where $PA = 2R\cos A$, $PB = 2R\cos B$, $PC = 2R\cos C$
- The distance of orthocentre from its sides BC, AC and AB are PL, PN and PM respectively, where $PL = 2R\cos B\cos C$, $PN = 2R\cos A\cos C$, $PM = 2R\cos A\cos B$ and 'R' is circumradius of the triangle.

m – *n* cot THEOREM OR *m* – *n* RULE

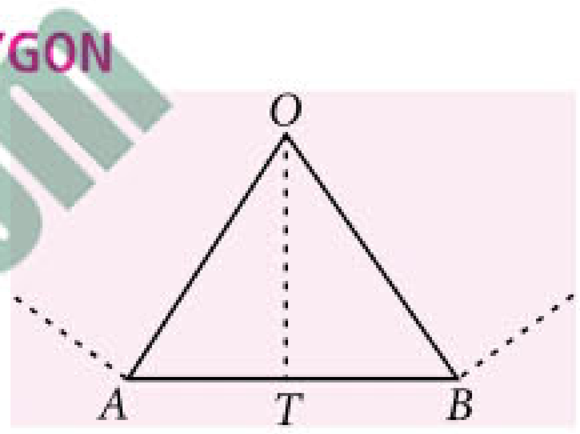
If D is a point on the side BC of a triangle ABC such that $BD : DC = m : n$, let $\theta = \angle ADC$, $\angle BAD = \alpha$, $\angle CAD = \beta$ then

- $(m+n)\cot\theta = m\cot\alpha - n\cot\beta$
- $(m+n)\cot\theta = n\cos\beta - m\cot\alpha$



CONCEPT OF REGULAR POLYGON

A polygon is said to be regular polygon if all its sides are equal in measure (length) and the measures of all the angles are equal.



- Let AB be a side of polygon and $OT = r$ (radius of incircle of polygon) $=\frac{a}{2}\cot\left(\frac{\pi}{n}\right)$ and $OA = OB = R$ (radius of circumcircle of polygon) $=\frac{a}{2}\text{cosec}\left(\frac{\pi}{n}\right)$
- Area of regular polygon of n sides inscribed in a circle of radius (r) = Number of sides × Area of

  $$\Delta OAB = n\times\frac{1}{2}(AB)\cdot OT = n\left(\frac{1}{2}\cdot 2AT\right)\cdot OT$$

  $$= n\cdot(AT\cdot OT)$$

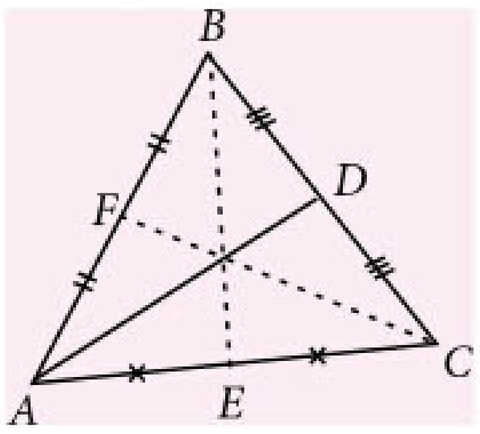
  $=\frac{na^2}{4}\cot\left(\frac{\pi}{n}\right)$ in terms of side of Δ

  $=nr^2\tan\left(\frac{\pi}{n}\right)$ in terms of in radius of the circle

  $=\frac{nR^2}{2}\sin\frac{2\pi}{n}$ in terms of circumradius of circle

- Sum of the radii of inscribed and circumscribed circles of n sided polygon $=r+R=\frac{a}{2}\cot\frac{\pi}{2n}$

LENGTH OF MEDIANS OF A TRIANGLE

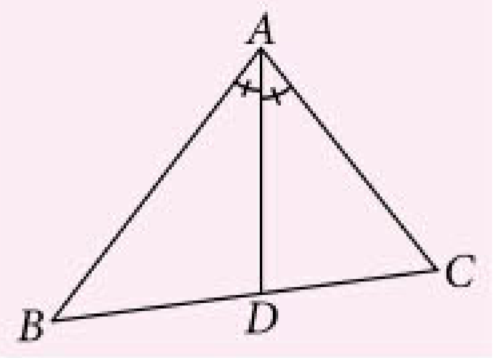


Median = AD (Drawn from A to side BC)

$$=\frac{1}{2}\sqrt{b^2+c^2+2bc\cos A}$$

$$=\frac{1}{2}\sqrt{2b^2+2c^2-a^2}$$ and so on

LENGTH OF ANGLE BISECTOR FROM THE ANGLE *A* OF TRIANGLE

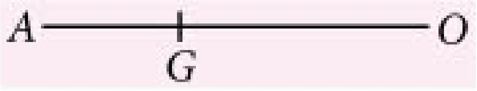


Length (AD), drawn from vertex A is given by

$$AD=\frac{2bc\cos\frac{A}{2}}{b+c}$$ and so on.

- Let O is orthocentre, G is centroid and A is circumcentre, then

(1) AO = Distance between circumcentre (A) and orthocentre (O)



$$=R\sqrt{1-8\cos A\cos B\cos C}$$

(2) AG = Distance between circumcentre and centroid

$$=\frac{1}{3}(AO)$$

(3) GO = Distance between centroid and orthocentre

$$\therefore\ GO=\frac{2}{3}AO=\frac{2}{3}R\sqrt{1-8\cos A\cos B\cos C}$$

- Let A is circumcentre and I is incentre and I_1, I_2, I_3 are excentres, then

(1) AI = Distance between circumcentre and incentre

$$=R\sqrt{1-8\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}=R\sqrt{1-\frac{2r}{R}}=\sqrt{R^2-2Rr}$$

$$=\sqrt{(R-r)^2-r^2}$$

(2) Distance between excentres I_1, I_2, I_3 and orthocentre

$$AI_1=R\sqrt{1+8\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}}$$

$$=R\sqrt{1+\frac{2r_1}{R}}=\sqrt{R(R+2r_1)}$$

$$AI_2=R\sqrt{1+8\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}}=\sqrt{R(R+2r_2)}$$

$$AI_3=R\sqrt{1+8\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}}=\sqrt{R(R+2r_3)}$$

- If $r_1=r_2+r_3+r$, $r_2=r_1+r_3+r$, $r_3=r_1+r_2+r$, then triangle is right angle triangle.

INVERSE TRIGONOMETRIC FUNCTIONS

- **Domain and Range of Inverse Trigonometric Functions**

Functions	Domain	Range
$\sin^{-1}x$	$-1\le x\le 1$	$\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$
$\cos^{-1}x$	$-1\le x\le 1$	$0\le \cos^{-1}x\le\pi$
$\tan^{-1}x$	R = set of real numbers = $(-\infty,\infty)$	$\frac{-\pi}{2}<\tan^{-1}x<\frac{\pi}{2}$
$\cot^{-1}x$	R = set of real numbers = $(-\infty,\infty)$	$0<\cot^{-1}x<\pi$
$\sec^{-1}x$	$R-(-1,1)$	$[0,\pi]-\left\{\frac{\pi}{2}\right\}$
$\mathrm{cosec}^{-1}x$	$R-(-1,1)$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$

Properties of Inverse Trigonometric Functions

- **Property 1 :** (1) $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}, -1\le x\le 1$

(2) $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}, x\in R$

(3) $\sec^{-1}x+\mathrm{cosec}^{-1}x=\frac{\pi}{2}, |x|\ge 1$

- **Property 2 :** (1) $\mathrm{cosec}^{-1}x=\sin^{-1}\left(\frac{1}{x}\right), |x|\ge 1$

(2) $\sec^{-1}x=\cos^{-1}\left(\frac{1}{x}\right), |x|\ge 1$

(3) $$\cot^{-1}x=\begin{cases}\tan^{-1}\left(\frac{1}{x}\right), & x>0\\ \pi+\tan^{-1}\left(\frac{1}{x}\right), & x<0\end{cases}$$

- **Property 3 :** The graph of $\sin^{-1}x$, $\tan^{-1}x$ and $\mathrm{cosec}^{-1}x$ are symmetrical about the origin.

(1) $\sin^{-1}(-x)=-\sin^{-1}(x)\ \forall\ x\in[-1,1]$

(2) $\tan^{-1}(-x)=-\tan^{-1}x\ \forall\ x\in R$

(3) $\mathrm{cosec}^{-1}(-x)=-\mathrm{cosec}^{-1}x\ \forall\ x\in(-\infty,-1]\cup[1,\infty)$

(4) $\cos^{-1}(-x)=\pi-\cos^{-1}x\ \forall|x|\le 1$

(5) $\sec^{-1}(-x)=\pi-\sec^{-1}x\ \forall|x|\ge 1$

(6) $\cot^{-1}(-x)=\pi-\cot^{-1}x\ \forall\ x\in R$

- **Property 4 :** (1) $\sin(\sin^{-1}x)=x, |x|\le 1$

(2) $\cos(\cos^{-1}x)=x, |x|\le 1$

(3) $\tan(\tan^{-1}x)=x, x\in R$

(4) $\cot(\cot^{-1}x)=x, x\in R$

(5) $\sec(\sec^{-1}x)=x, x\in R-(-1,1)$

(6) $\mathrm{cosec}(\mathrm{cosec}^{-1}x)=x, x\in R-(-1,1)$

- **Property 5 :** (1)

$$\sin^{-1}(\sin x)=\begin{cases} x, \forall\, x\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right] \\ \pi-x, \forall\, x\in\left[\frac{\pi}{2},\frac{3\pi}{2}\right] \\ x-2\pi\ \ \forall\, x\in\left[\frac{3\pi}{2},\frac{5\pi}{2}\right] \\ 3\pi-x\ \ \forall\ \ x\ \in\left[\frac{5\pi}{2},\frac{7\pi}{2}\right] \end{cases}$$

Period of $\sin^{-1}(\sin x)$ is 2π.

(2) $$\cos^{-1}(\cos x)=\begin{cases} x, \forall\, x\in[0,\pi] \\ 2\pi-x,\ \forall\ \ x\in[\pi,2\pi] \\ x-2\pi, \forall\, x\in[2\pi,3\pi] \\ 4\pi-x,\ \forall\ \ x\in[3\pi,4\pi] \end{cases}$$

Period of $\cos^{-1}$ ($\cos x$) is 2π.

(3) $$\tan^{-1}(\tan x)=\begin{cases} x,\ \forall\ \ x\ \in\left(\frac{-\pi}{2},\frac{\pi}{2}\right) \\ x-\pi,\ \forall\ \ x\in\left(\frac{\pi}{2},\frac{3\pi}{2}\right) \\ x-2\pi, \forall\, x\in\left(\frac{3\pi}{2},\frac{5\pi}{2}\right) \end{cases}$$

The function $\tan^{-1}(\tan x)$ is periodic with period π

(4) $\cot^{-1}(\cot x) = \{-n\pi + x,\ \forall\ \ x\in (n\pi, (n + 1)\pi), n\in Z$

- **Property 6 :** (1) $\sin^{-1}x + \sin^{-1}y =$

$$=\begin{cases} \sin^{-1}(x\sqrt{1-y^2}+y\sqrt{1-x^2})\text{ if } x\geq 0, y\geq 0, (x^2+y^2)\leq 1 \\ \pi-\sin^{-1}(x\sqrt{1-y^2}+y\sqrt{1-x^2})\text{ if } x\geq 0, y\geq 0, x^2+y^2>1 \\ -\pi-\sin^{-1}(x\sqrt{1-y^2}+y\sqrt{1-x^2})\text{ if } x, y<0 \end{cases}$$

(2) $\sin^{-1}x - \sin^{-1}y$

$$=\begin{cases} \sin^{-1}(x\sqrt{1-y^2}-y\sqrt{1-x^2})\text{ if } x\geq 0,\ y\leq 1 \\ \qquad\qquad i.e., x, y\in[0,1] \end{cases}$$

(3) $\cos^{-1}x + \cos^{-1}y$

$$=\begin{cases} \cos^{-1}(xy-\sqrt{1-x^2}\sqrt{1-y^2}\text{ if } x\geq 0,\ y\leq 1 \\ 2\pi-\cos^{-1}(xy-\sqrt{1-x^2}.\sqrt{1-y^2}\text{ if } x\geq -1,\ y<1 \end{cases}$$

(4) $\cos^{-1}x\ -\cos^{-1}y$

$$=\begin{cases} \cos^{-1}(xy+\sqrt{1-x^2}\cdot\sqrt{1-y^2}), \text{if } 0\leq x<y\leq 1 \\ -\cos^{-1}(xy+\sqrt{1-x^2}\cdot\sqrt{1-y^2}, \text{if } 0\leq y<x\leq 1 \end{cases}$$

(5) $\tan^{-1}x + \tan^{-1}y$

$$=\begin{cases} \tan^{-1}\frac{x+y}{1-xy}\text{ if } x, y\geq 0, xy<1\ i.e., 0\leq xy<1 \\ \pi+\tan^{-1}\left(\frac{x+y}{1-xy}\right)\text{if } x>0, y>0, xy>1 \end{cases}$$

(6) $\tan^{-1}x - \tan^{-1}y$

$$=\begin{cases} \tan^{-1}\left(\frac{x-y}{1+xy}\right), x>0, y>0\ i.e., xy>-1 \end{cases}$$

- **Property 7 :**

(1) $$\sin^{-1}(2x\sqrt{1-x^2})=\begin{cases} 2\sin^{-1}x, & \text{if } -\frac{1}{\sqrt{2}}\leq x\leq\frac{1}{\sqrt{2}} \\ \pi-2\sin^{-1}x, & \text{if } \frac{1}{\sqrt{2}}<x\leq 1 \\ -(\pi+2\sin^{-1}x)\text{if } -1\leq x\leq-\frac{1}{\sqrt{2}}, \end{cases}$$

(2) $$\cos^{-1}(2x^2-1)=\begin{cases} 2\pi-2\cos^{-1}x & \text{if } -1\leq x<0 \\ 2\cos^{-1}x & \text{if } 0\leq x\leq 1 \end{cases}$$

(3) $$\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\begin{cases} \pi+2\tan^{-1}x, \text{if } x<-1 \\ 2\tan^{-1}x, \text{if } -1<x<1 \\ -\pi+2\tan^{-1}x, \text{if } x>1 \end{cases}$$

(4) $$\sin^{-1}\left(\frac{2x}{1+x^2}\right)=\begin{cases} -(\pi+2\tan^{-1}x) & \text{if } x<-1 \\ 2\tan^{-1}x & \text{if } -1\leq x\leq 1 \\ \pi-2\tan^{-1}x & \text{if } x>1 \end{cases}$$

(5) $$\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)=\begin{cases} -2\tan^{-1}x & \text{if } x<0 \\ 2\tan^{-1}x & \text{if } x\geq 0 \end{cases}$$

(6) $\cos^{-1}(4x^3 - 3\cos x)$

$$=\begin{cases} 3\cos^{-1}x, & \frac{1}{2}\leq x\leq 1 \\ 2\pi-3\cos^{-1}x, & |x|\leq\frac{1}{2}\ i.e., \frac{-1}{2}\leq x\leq\frac{1}{2} \\ 3\cos^{-1}x-2\pi, & -1\leq x\leq-\frac{1}{2} \end{cases}$$

(7) $$3\tan^{-1}x=\begin{cases} \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), & \text{if } |x|<\frac{1}{\sqrt{3}} \\ \pi+\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), & \text{if } x>\frac{1}{\sqrt{3}} \\ -\pi+\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), & \text{if } x<-\frac{1}{\sqrt{3}} \end{cases}$$

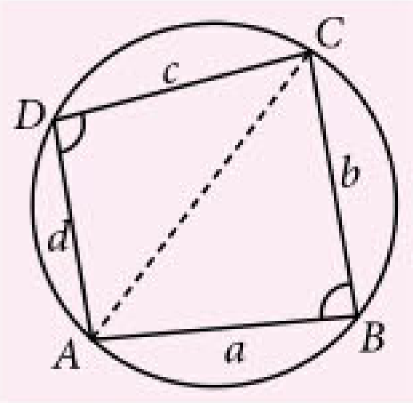
(8) $$3\sin^{-1}x=\begin{cases}\sin^{-1}(3x-4x^3), \text{ if } x\in\left[\dfrac{-1}{2},\dfrac{1}{2}\right]\\ \pi-\sin^{-1}(3x-4x^3), \text{ if } x\in\left[\dfrac{1}{2},1\right]\\ -\pi-\sin^{-1}(3x-4x^3), \ x\in\left[-1,-\dfrac{1}{2}\right]\end{cases}$$

- If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\dfrac{3\pi}{2}$
  then $\sin^{-1}x=\sin^{-1}y=\sin^{-1}z=\dfrac{\pi}{2}$ so $x=y=z=1$
- If $\cos^{-1}x+\cos^{-1}y=\dfrac{\pi}{2}$, $\sin^{-1}x+\sin^{-1}y=\dfrac{\pi}{2}$
  then $x^2 + y^2 = 1$ for both cases.
- If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$ then $x + y + z = xyz$
- If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\dfrac{\pi}{2}$
  then $xy + yz + zx = 1$
- If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\dfrac{\pi}{2}$
  then $x^2 + y^2 + z^2 + 2xyz = 1$ and $x=y=z=\dfrac{1}{2}$
  $\because\ \sin^{-1}x=\sin^{-1}y=\sin^{-1}z=\dfrac{\pi}{6}$
- If $a\sin^{-1}x - b\cos^{-1}x = c$ then
  $a\sin^{-1}x+b\cos^{-1}x=\dfrac{\pi ab+c(a-b)}{a+b}$
- (i) $\sin^{-1}x + \sin^{-1}y = \pi \Leftrightarrow x = y = 1$ and
  (ii) $\sin^{-1}x + \sin^{-1}y = -\pi \Leftrightarrow x = y = -1$
- (i) $\cos^{-1}x + \cos^{-1}y = 0 \Leftrightarrow x = y = 1$
  (ii) $\cos^{-1}x + \cos^{-1}y = 2\pi \Leftrightarrow x = y = -1$
- $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3$
  $=\dfrac{\pi}{4}+\left(\pi+\tan^{-1}\left(\dfrac{2+3}{1+2.3}\right)\right)=\dfrac{\pi}{4}+\dfrac{3\pi}{4}=\pi$

CONCEPT OF CYCLIC QUADRILATERAL

A quadrilateral is said to be cyclic quadrilateral if all its vertices lies on a circle.

- If A, B, C, D are vertices of a cyclic quadrilateral taken in order then
  (1) $\cos A + \cos B + \cos C + \cos D = 0$
  (2) $\cos(180° - A) + \cos(180° - B) + \cos(180° - C) + \cos(180° - D) = 0$
  (3) $\angle A + \angle C = \angle B + \angle D = 180°$
- $ABCD$ is a cyclic quadrilateral whose sides are $AB = a$, $BC = b$, $CD = c$, $DA = d$
  $\therefore$ Area of cyclic quadrilateral $= ar\Delta ABC + ar\Delta ACD$



$$=\frac{1}{2}ab\sin B+\frac{1}{2}cd\sin D$$

$$=\frac{1}{2}(ab+cd)\sin B \quad (\because\ \angle D=180°-\angle B)$$

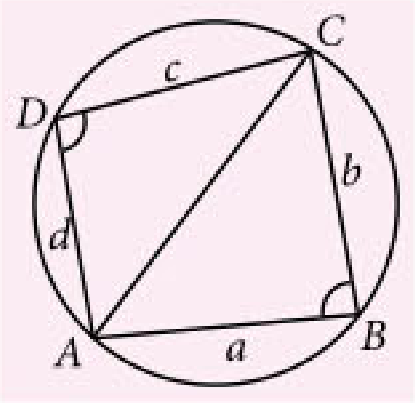
$$=\sqrt{(s-a)(s-b)(s-c)(s-d)}$$

- $\cos B=\dfrac{(a^2+b^2)-(c^2+d^2)}{2(ab+cd)}$ ...(1)
- $\cos A=\dfrac{(a^2+d^2)-(b^2+c^2)}{2(ad+bc)}$
- $\cos C=\dfrac{(b^2+c^2)-(a^2+d^2)}{2(bc+ad)}$
- $\cos D=\dfrac{(c^2+d^2)-(a^2+b^2)}{2(cd+ab)}$

CIRCUMRADIUS OF A CYCLIC QUADRILATERAL

From the figure, we notice that the circumcircle of quadrilateral $ABCD$ and triangle ACB are same.

$\therefore$ R = circumradius of cyclic quadrilateral $ABCD$ = circumradius of ΔACB



$$\therefore\ R=\frac{AC}{2\sin B}=\frac{AC}{2\times\left(\dfrac{2\text{ Area of cyclic quadrilateral }ABCD}{ab+cd}\right)}$$

$$R=\frac{AC(ab+cd)}{4\cdot\text{Area of cyclic quadrilateral}}$$

$$=\sqrt{\frac{(ac+bd)(ad+bc)}{ab+cd}}\times\frac{ab+cd}{4\Delta}$$

$$=\frac{1}{4}\sqrt{\frac{(ac+bd)(ad+bc)(ab+cd)}{(s-a)(s-b)(s-c)(s-d)}}$$

Using $AC=\sqrt{\dfrac{(ac+bd)(ad+bc)}{ab+cd}}$

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JEE PRACTICE PAPER 2018 ADVANCED

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SINGLE OPTION CORRECT TYPE

1. The function $y = x - \cot^{-1} x - \log\left(x - \sqrt{x^2+1}\right)$ is increasing on

(a) $(0, 0)$ (b) $(-\infty, 0)$ (c) $(0, \infty)$ (d) $(-\infty, \infty)$

2. An infinite G. P. has 2nd term x and its sum is 4, then x belongs to

(a) $(0, 2]$ (b) $(1, 8)$

(c) $(-8, 1]$ (d) none of these

3. $\sum_{r=0}^{10} 2^r \, {}^{20}C_r \, {}^{20-r}C_{10-r}$ is equal to

(a) ${}^{20}C_{10}$ (b) ${}^{20}C_{10}\left(\frac{3}{2}\right)^{10}$

(c) ${}^{20}C_{10}\, 3^{10}$ (d) ${}^{20}C_{10}\, 2^{10}$

4. If the graphs of the functions $y = \ln x$ and $y = ax$ intersect at exactly two points, then a must be

(a) $(0, e)$ (b) $(1/e, 0)$

(c) $(0, 1/e)$ (d) none of these

5. If $\int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} = \frac{\pi}{2ab}$, then the value of $\int_0^{\pi/2} \frac{dx}{(4\cos^2 x + 9\sin^2 x)^2}$ is equal to

(a) $\frac{11\pi}{864}$ (b) $\frac{13\pi}{864}$

(c) $\frac{17\pi}{864}$ (d) none of these

6. If $(3 + x^{2008} + x^{2009})^{2010} = a_0 + a_1x + a_2x^2 + ... + a_nx^n$, then the value of $a_0 - \frac{1}{2}a_1 - \frac{1}{2}a_2 + a_3 - \frac{1}{2}a_4 - \frac{1}{2}a_5 + a_6 - ...$ is

(a) 3^{2010} (b) 1

(c) 2^{2010} (d) none of these

7. Let AB be any chord of the circle $x^2 + y^2 - 4x - 4y + 4 = 0$ which subtends an angle of 90° at the point (2, 3), then the locus of the mid point of AB is circle whose centre is

(a) $(1, 5)$ (b) $(1, 5/2)$

(c) $(1, 3/2)$ (d) $(2, 5/2)$

8. If k be a positive integer, then the number of values of k satisfying

$$\int_0^{\pi/2}\left[k^2\left(\cos 3x + \frac{1}{2}\cos x\right) + \sin x - 2k\cos x\right]dx \le 1, \text{ is}$$

(a) 10 (b) 11

(c) 12 (d) none of these

9. The solution of differential equation $(x\cot y + \ln\cos x)dy + (\ln\sin y - y\tan x)dx = 0$ is

(a) $(\sin x)^y (\cos y)^x = c$ (b) $(\sin y)^x (\cos x)^y = c$

(c) $(\sin x)^y (\sin y)^x = c$ (d) $(\cot x)^y (\cot y)^x = c$

10. The value of $\sum_{r=0}^{n-1}\left(\frac{n+1}{n}\right)\left(\frac{r.{}^nC_r\,{}^nC_{r+1}}{r+2}\right)$ is

(a) ${}^{2n-1}C_{n+1}$ (b) ${}^{2n}C_{n-1}$

(c) ${}^{2n-1}C_{n-1}$ (d) ${}^{2n}C_{n-2}$

11. The number of points in the cartesian plane with integral co-ordinates satisfying the inequations $|x| \le 10$, $|y| \le 10$ and $|x - y| \le 10$ is

(a) 321 (b) 331

(c) 341 (d) none of these

12. If $f(x + y) = f(x) - f(y) + 2xy - 1 \; \forall \; x, y \in R$. Also if $f(x)$ is differentiable and $f'(0) = b$ also $f(x) > 0 \; \forall \; x$, then the set of values of b is

(a) ϕ (b) $\{1\}$

(c) $\{1, 2\}$ (d) none of these

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13. Let $a_1 = \left(\tan\frac{\pi}{8}\right)^{\tan\frac{\pi}{8}}$, $a_2 = \left(\tan\frac{\pi}{8}\right)^{\cot\frac{\pi}{8}}$, $a_3 = \left(\cot\frac{\pi}{8}\right)^{\cot\frac{\pi}{8}}$, $a_4 = \left(\cot\frac{\pi}{8}\right)^{\tan\frac{\pi}{8}}$, then which of the following statement(s) is/are true?

(a) $a_4 > a_3 > a_2 > a_1$
(b) $a_3 > a_4 > a_2 > a_1$
(c) $a_3 > a_4 > a_1 > a_2$
(d) $a_3 > a_1 > a_2 > a_4$

14. The equation of curve passing through (1, 1) in which the subtangent is always bisected at the origin, is

(a) $y^2 = x$
(b) $2x^2 - y^2 = 1$
(c) $x^2 + y^2 = 2$
(d) $x + y = 2$

15. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors such that $\vec{b} \times \vec{c} = \vec{a}$, $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{c} \times \vec{a} = \vec{b}$ then

(a) $[\vec{a}\ \vec{b}\ \vec{c}] = 1$
(b) $[\vec{a}\ \vec{b}\ \vec{c}] \neq 1$
(c) $|\vec{a}| + |\vec{b}| + |\vec{c}| = 5$
(d) none of these

16. If in a ΔABC, $\cos A + 2\cos B + \cos C = 2$, then a, b, c are in

(a) A.P.
(b) H.P.
(c) G.P.
(d) A.G.P.

17. The value of $a (a < 0)$ for which least value of quadratic expression $4x^2 - 4ax + a^2 - 2a + 2$ on the interval $0 \le x \le 2$ is equal to 3, is

(a) $-\sqrt{2}$
(b) $-\sqrt{2} - 2$
(c) $1 - \sqrt{2}$
(d) none of these

18. If $f(x) = \tan\frac{3\pi}{2}[x] + \cos^2 \pi[x] + e^{\left\{\frac{x}{3}\right\}}$ (where [.] denotes the greatest integer function and {.} denotes fractional part), then

(a) $f(x)$ is periodic with fundamental period not defined
(b) $f(x)$ is periodic with period 6
(c) $f(x)$ is non periodic
(d) $f(x)$ is periodic with period 2

19. If the shortest distance between lines
$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda_1(2\hat{i} + 3\hat{j} + 4\hat{k})$ and
$\vec{r} = 2\hat{i} + 4\hat{j} + 5\hat{k} + \lambda_2(3\hat{i} + 4\hat{j} + 5\hat{k})$ is x.
Then $\cos^{-1}\cos\sqrt{6}\,x$ is equal to

(a) 1/2
(b) 0
(c) 1
(d) π

20. The maximum values of $\frac{\log x}{x}$ is

(a) 1
(b) $2/e$
(c) e
(d) $1/e$

21. The maximum value of $\cos(\sin x) + \sin(\cos x)$, $0 \le x \le \pi$, is

(a) 1 + sin 1
(b) cos 1
(c) 1 + cos 1
(d) $2\sin^2\frac{1}{2}$

22. The sides of the triangle are in the ratio $1 : \sqrt{3} : 2$, then the corresponding angles of triangle are in the ratio

(a) 1 : 3 : 4
(b) 2 : 3 : 4
(c) 3 : 2 : 1
(d) 1 : 2 : 3

ONE OR MORE THAN ONE OPTION CORRECT TYPE

23. $ABCD$ is a cyclic quadrilateral then the correct statements are

(a) if B lies inside the circle through A, C and D, then D lies inside the circle through A, B and C.
(b) if B lies outside the circle through A, C and D, then D lies outside the circle through A, B and C.
(c) if B lies inside the circle through A, C and D, then D lies outside the circle through A, B and C.
(d) if B lies outside the circle through A, C and D, then D lies inside the circle through A, B and C.

24. Triangles $A_1A_2A_3$ and $B_1B_2B_3$ have side lengths (a_1, a_2, a_3) and (b_1, b_2, b_3) respectively satisfying the relation

$$\sqrt{a_1 + a_2 + a_3}\sqrt{b_1 + b_2 + b_3} = \sqrt{a_1b_1} + \sqrt{a_2b_2} + \sqrt{a_3b_3},$$

then which one of the following statements is/are true ?

(a) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$
(b) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} = 1$
(c) $\Delta A_1A_2A_3$ and $\Delta B_1B_2B_3$ are similar
(d) $\Delta A_1A_2A_3$ and $\Delta B_1B_2B_3$ are congruent

25. If the equation $\bar{a}z + a\bar{z} + b = 0$ and $\bar{a}z - a\bar{z} + b_1 = 0$ represent two lines L_1 and L_2 in the complex plane then

(a) L_1 and L_2 are perpendicular
(b) b is purely real
(c) b_1 is purely imaginary
(d) b_1 is purely real

26. Let $f : R \to R$ be a function defined by $f(x+1) = \frac{f(x) - 5}{f(x) - 3}\ \forall \in R$. Then which of the following statement(s) is/are true?

(a) $f(2008) = f(2004)$
(b) $f(2006) = f(2010)$
(c) $f(2006) = f(2002)$
(d) $f(2006) = f(2018)$

27. If the equation $\sec x + \operatorname{cosec} x = k$, $(k \in R)$ has exactly 3 solutions between 0 and π, then k can take values

(a) $\sqrt{2}$
(b) 2
(c) $3\sqrt{2}$
(d) 4

COMPREHENSION TYPE

Let $\vec{r}$ is a position vector of a variable point in cartesian OXY plane such that $\vec{r}\cdot(10\hat{j}-8\hat{i}-\vec{r})=40$ and $p_1=\max\{|\vec{r}+2\hat{i}-3\hat{j}|^2\}$, $p_2=\min\{|\vec{r}+2\hat{i}-3\hat{j}|^2\}$.

A tangent line is drawn to the curve $y=\frac{8}{x^2}$ at the point A with abscissa 2. The drawn line cuts x-axis at a point B.

28. p_2 is equal to
(a) 9 (b) $2\sqrt{2}-1$
(c) $6\sqrt{2}+3$ (d) $9-4\sqrt{2}$

29. p_1+p_2 is equal to
(a) 2 (b) 10 (c) 18 (d) 5

30. $\overrightarrow{AB}\cdot\overrightarrow{OB}$ is
(a) 1 (b) 2 (c) 3 (d) 4

MATRIX-MATCH TYPE

31. Match the following :

Column-I		Column-II	
(A)	If $y=\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}(b)$, $(0 < b < 1)$ and $0 < y \le \pi/4$, then the maximum value of b will be	(1)	3
(B)	The number of solutions of $\sin^4 x + \cos^3 x \ge 1$ in $(0, 2\pi)$ will be	(2)	1/3
(C)	The area bounded by the loop of $4y^2 = x^2(4 - x^2)$ is	(3)	1/2
(D)	If $(1+x^2)\frac{dy}{dx}+2xy-1=0$ passes through origin, then $y(1)$ will be	(4)	16/3

32. Match the following :

Column-I		Column-II	
(A)	The differential equation $y=x\frac{dy}{dx}+\frac{dx}{dy}$ represents	(1)	pair of straight lines
(B)	If an ellipse slides between two perpendicular straight lines, then the locus of its centre is	(2)	straight line
(C)	If a, b are real numbers and $c > 0$, then the locus of (x,y) represented by the equation $\|ay-bx\|=c\sqrt{(x-a)^2+(y-b)^2}$ is	(3)	circle
(D)	If $h \neq 0$ and $\frac{a}{h}=\frac{\sqrt{2}c}{b}$, then the equation $hxy + ax + by + c = 0$ represents	(4)	parabola
		(5)	hyperbola

33. Match the following :

Column-I		Column-II	
(A)	$\vec{a}$, $\vec{b}$ are unit vectors and $\vec{a}+2\vec{b}\perp 5\vec{a}-4\vec{b}$, then $2(\vec{a}\cdot\vec{b})$ is equal to	(1)	0
(B)	The points (1, 0, 3), (–1, 3, 4), (1, 2, 1), (k, 2, 5) are coplanar, then k =	(2)	–1
(C)	The vectors (1, 1, m), (1, 1, m + 1), (1, –1, m) are coplanar, then number of values of m is	(3)	1
(D)	$\vec{a}\times(\vec{b}\times\vec{c})+\vec{b}\times(\vec{c}\times\vec{a})+\vec{c}\times(\vec{a}\times\vec{b})$ is equal to	(4)	2

34. Match the following :

Column-I		Column-II	
(A)	If the line $\frac{x-1}{1}=\frac{y+1}{-2}=\frac{z+1}{\lambda}$ lies in the plane $3x-2y+5z=0$, then x is equal to	(1)	$\sin^{-1}\sqrt{\frac{6}{25}}$
(B)	If $(3, \lambda, \mu)$ is a point on the line $2x + y + z - 3 = 0 = x - 2y + z - 1$, then $\lambda + \mu$ is equal to	(2)	–7/5
(C)	The angle between the line $x = y = z$ and the plane $4x - 3y + 5z = 2$ is	(3)	–3
(D)	The angle between the planes $x + y + z = 0$ and $3x - 4y + 5z = 0$ is	(4)	$\cos^{-1}\sqrt{\frac{8}{75}}$

35. Match the following :

Column-I		Column-II	
(A)	The number of arrangements of the word BANANA in which two N's do not appear together is	(1)	[1, 2)
(B)	If $(1+x)^n=C_0+C_1x+...+C_nx^n$, then $\left(1+\frac{C_1}{C_0}\right)...\left(1+\frac{C_n}{C_{n-1}}\right)$ is	(2)	$\frac{(n+1)^n}{n!}$

(C)	If c, d, e are in A.P., then $\Delta = \begin{vmatrix} x+1 & x+2 & x+c \\ x+2 & x+3 & x+d \\ x+3 & x+4 & x+e \end{vmatrix}$ is	(3)	$\frac{2(n+1)^n}{n!}$
(D)	$\frac{c^2+d^2+e^2}{cd+de+ce}$ lies between (where c, d, e are the lengths of the sides of ΔCDE)	(4)	0
		(5)	40

INTEGER ANSWER TYPE

36. If $f(x+y+z) = f(x) + f(y) + f(z)$ with $f(1) = 1$ and $f(2) = 2$ and $x, y, z \in N$, then $\lim\limits_{n\to\infty} \frac{\sum_{r=1}^{n}(4r)f(3r)}{n^3}$ is equal to ________.

37. $x, y \in R$, $x^2 + y^2 + xy = 1$, then the minimum value of $x^3y + xy^3 + 4$ is _____.

38. If the equation of the curve on reflection of the ellipse $\frac{(x-4)^2}{16} + \frac{(y-3)^2}{9} = 1$ about the line $x - y - 2 = 0$ is $16x^2 + 9y^2 + k_1x - 36y + k_2 = 0$, then $\frac{k_1+k_2}{44}$ is ______.

39. If the number of ordered pairs of (x, y) satisfying the system of equations $5x\left(1+\frac{1}{x^2+y^2}\right) = 12$ and $5y\left(1-\frac{1}{x^2+y^2}\right) = 4$ is n, then n is ______.

40. If x, y, z are distinct positive numbers such that $x + \frac{1}{y} = y + \frac{1}{z} = z + \frac{1}{x}$, then the value of xyz is ______.

SOLUTIONS

1. (d) : $y = x - \cot^{-1}x - \log\left(x - \sqrt{x^2+1}\right)$,

$$\frac{dy}{dx} = 1 + \frac{1}{1+x^2} + \frac{1}{\sqrt{x^2+1}} \text{ as } \frac{dy}{dx} > 0 \text{ for all } R,$$

So, y is increasing on $(-\infty, \infty)$.

2. (c) : Sum$= \frac{\frac{x}{r}}{1-r} = 4$ (where r is common ratio)

$x = 4r(1 - r) = 4(r - r^2)$

For $r \in (-1, 1)$, $r - r^2 \in \left(-2, \frac{1}{4}\right]$ $\Rightarrow x \in (-8, 1]$

3. (c) : $\sum_{r=0}^{10} 2^r\, {}^{20}C_r\, {}^{20-r}C_{10-r}$

$$= \sum_{r=0}^{10} 2^r \frac{20!}{r!(20-r)!} \times \frac{(20-r)!}{10!\,(10-r)!}$$

$$= \sum_{r=0}^{10} 2^r \frac{20!}{10!10!} \times \frac{10!}{r!(10-r)!} = \sum_{r=0}^{10} 2^r\, {}^{20}C_{10}\, {}^{10}C_r = {}^{20}C_{10}\, 3^{10}$$

4. (c) : Given curves are $y = \ln x$ and $y = ax$

$\Rightarrow$ $\ln x = ax$ has exactly two solutions.

$\Rightarrow$ $\frac{\ln x}{x} = a$ has exactly two solutions.

Let $y = \frac{\ln x}{x}$, $x > 0$, $\frac{dy}{dx} = \frac{x \cdot \frac{1}{x} - \ln x}{x^2} = \frac{1-\ln x}{x^2}$

y is increasing if $1 - \ln x > 0$, $\ln x < 1$, $0 < x < e$, range of y is $\left(-\infty, \frac{1}{e}\right]$. Thus, for exactly two solutions of $\frac{\ln x}{x} = a$, $a \in \left(0, \frac{1}{e}\right)$

5. (b) : Let $I = \int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} = \frac{\pi}{2ab}$

Differentiating w.r.t. a, we get

$$\int_0^{\pi/2} \frac{-dx 2a\cos^2 x}{(a^2\cos^2 x + b^2\sin^2 x)^2} = -\frac{\pi}{2a^2b}$$

$$\Rightarrow \int_0^{\pi/2} \frac{\cos^2 x dx}{(a^2\cos^2 x + b^2\sin^2 x)^2} = \frac{\pi}{4a^3b} \quad \text{...(i)}$$

Similarly, differentiating w.r.t. b, we get

$$\int_0^{\pi/2} \frac{\sin^2 x dx}{(a^2\cos^2 x + b^2\sin^2 x)^2} = \frac{\pi}{4ab^3} \quad \text{...(ii)}$$

Adding (i) & (ii), we get

$$\int_0^{\pi/2} \frac{dx}{(a^2\cos^2 x + b^2\sin^2 x)^2} = \frac{\pi}{4ab}\left[\frac{1}{a^2} + \frac{1}{b^2}\right]$$

$$\therefore \int_0^{\pi/2} \frac{dx}{(4\cos^2 x + 9\sin^2 x)^2} = \frac{\pi}{24}\left[\frac{1}{4} + \frac{1}{9}\right] = \frac{13\pi}{864}$$

6. (c) : Putting $x = \omega, \omega^2$, we get

$(3 + \omega + \omega^2)^{2010} = a_0 + a_1\omega + a_2\omega^2 + ...$

$$\Rightarrow \quad 2^{2010} = a_0 + a_1\omega + a_2\omega^2 + a_3 + a_4\omega + ... \quad \text{...(i)}$$

$$\text{and } 2^{2010} = a_0 + a_1\omega^2 + a_2\omega + a_3 + a_4\omega^2 + ... \quad \text{...(ii)}$$

Adding (i) and (ii), we get

$$2^{2011} = 2a_0 - a_1 - a_2 + 2a_3 - a_4 - a_5 + 2a_6 - ...$$

$$2^{2010} = a_0 - \frac{1}{2}a_1 - \frac{1}{2}a_2 + a_3 - \frac{1}{2}a_4 - \frac{1}{2}a_5 + a_6 - ....$$

7. (d) : Let mid point of AB is $M(h, k)$.
AB subtends 90° at (2, 3).

$\Rightarrow \; AM = MB = \sqrt{(h-2)^2 + (k-3)^2}$
Also, $CM^2 + MB^2 = CB^2$
$\Rightarrow \; (h - 2)^2 + (k - 2)^2 + (h - 2)^2 + (k - 3)^2 = 4$

$$\Rightarrow \quad x^2 + y^2 - 4x - 5y + \frac{17}{2} = 0$$

8. (c) : $\left[k^2\left(\frac{\sin 3x}{3} + \frac{1}{2}\sin x\right) - \cos x - 2k\sin x\right]_0^{\pi/2} \le 1$

$$\Rightarrow \quad \left\{k^2\left(\frac{-1}{3} + \frac{1}{2}\right) - 0 - 2k\right\} - \{-1\} \le 1 \Rightarrow \frac{k^2}{6} - 2k + 1 \le 1$$

$\Rightarrow \; k(k - 12) \le 0, \; 0 \le k \le 12, \; k = 1, 2, 3, 4, ...,12.$

9. (b) : $(x\cot y + \ln\cos x)dy + (\ln\sin y - y\tan x)dx = 0$
$\Rightarrow \; x\cot y \, dy + \ln\sin y \, dx + \ln\cos x \, dy - y\tan x \, dx = 0$
$\Rightarrow \; \int d(x\ln\sin y) + \int d(y\ln\cos x) = 0$
$\Rightarrow \; x\ln\sin y + y\ln\cos x = \ln k \; \Rightarrow \; (\sin y)^x(\cos x)^y = c$

10. (d) : $\sum_{r=0}^{n-1}\left(\frac{n+1}{n}\right)\left(\frac{r.\,{}^nC_r\,{}^nC_{r+1}}{r+2}\right) = \sum_{r=0}^{n-1} {}^{n-1}C_{r-1}\,{}^{n+1}C_{r+2}$

$$= {}^{n-1}C_0\,{}^{n+1}C_3 + \ldots + {}^{n-1}C_{n-2}\,{}^{n+1}C_{n+1}$$

$(1+x)^{n-1} = {}^{n-1}C_0 + {}^{n-1}C_1x + {}^{n-1}C_2x^2 + \ldots + {}^{n-1}C_{n-1}x^{n-1}$
$(1+x)^{n+1} = {}^{n+1}C_0x^{n+1} + {}^{n+1}C_1x^n + {}^{n+1}C_2x^{n-1} + \ldots + {}^{n+1}C_{n+1}$

Coefficient of x^{n-2} in $(1+x)^{2n} = {}^{2n}C_{n-2}$.

11. (b) : $|x| \le 10 \;\; \Rightarrow \;\; -10 \le x \le 10,$
$|y| \le 10 \;\; \Rightarrow \;\; -10 \le y \le 10,$
$|x - y| \le 10 \;\; \Rightarrow \;\; -10 \le x - y \le 10$
Total number of points
$= (20 + 1)^2 - 2[1 + 2 + ... + 10] = 331$

12. (a) : $f(0) = f(0) - f(0) + 2(0) - 1 = -1$

Now, $f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$

$$= \lim_{h\to 0}\frac{f(x) - f(h) + 2xh - 1 - f(x)}{h}$$

$$= \left\{\lim_{h\to 0}\frac{-f(h) + f(0)}{h}\right\} + 2x \Rightarrow f'(x) = 2x - f'(0) = 2x - b$$

$\therefore \;\; f(x) = x^2 - bx + c$
But $f(0) = -1$
$\Rightarrow \; f(x) = x^2 - bx - 1, \; D = b^2 + 4 > 0 \; \forall \; b \in R$
Hence, $f(x)$ cannot be > 0 always $\forall \; x \in R$.

13. (c) : As $0 < \frac{\pi}{8} < \frac{\pi}{4} \;\; \Rightarrow \; \cot\frac{\pi}{8} > 1 > \tan\frac{\pi}{8}$
$\Rightarrow \; a_3 > a_4$ and $a_1 > a_2$

Again, $\cot\frac{\pi}{8} > 1 > \tan\frac{\pi}{8} > 0 \;\; \Rightarrow a_1 < a_3$ and $a_1 < a_4$
Hence, $a_2 < a_1 < a_4 < a_3$.

14. (a) : TM is subtangent where $T \equiv \left(x - y\frac{dx}{dy}, 0\right)$
and $M \equiv (x, 0)$, $\frac{1}{2}\left(x - y\frac{dx}{dy} + x\right) = 0$

$$\Rightarrow \quad 2x - y\frac{dx}{dy} = 0 \quad \Rightarrow \quad 2\int\frac{dy}{y} = \int\frac{dx}{x}$$

$\Rightarrow \; 2\ln y = \ln cx \Rightarrow y^2 = cx$, curve passes through (1, 1).
$\therefore \;\; c = 1, \; y^2 = x$

15. (a) : $\vec{b}\times\vec{c} = \vec{a} \;\; \Rightarrow \;\; [\vec{a}\,\vec{b}\,\vec{c}] = \vec{a}\cdot\vec{a} = |\vec{a}|^2$
Similarly, $\vec{a}\times\vec{b} = \vec{c} \;\; \Rightarrow [\vec{a}\,\vec{b}\,\vec{c}] = |\vec{c}|^2$,
And $\vec{c}\times\vec{a} = \vec{b} \;\; \Rightarrow \;\; [\vec{a}\,\vec{b}\,\vec{c}] = |\vec{b}|^2$
Also, $[\vec{a}\,\vec{b}\,\vec{c}] = [\vec{b}\times\vec{c} \;\; \vec{c}\times\vec{a} \;\; \vec{a}\times\vec{b}] = [\vec{a}\,\vec{b}\,\vec{c}]^2 \Rightarrow [\vec{a}\,\vec{b}\,\vec{c}] = 1$

16. (a) : $\cos A + \cos C = 2(1 - \cos B)$

$$\Rightarrow \quad 2\cos\frac{A+C}{2}\cos\frac{A-C}{2} = 2\cdot 2\sin^2\frac{B}{2},$$

$$\Rightarrow \quad 2\cos\frac{B}{2}\cos\frac{A-C}{2} = 2\left(2\sin\frac{B}{2}\cos\frac{B}{2}\right) = 2\sin B$$

$$\Rightarrow \quad 2\sin\frac{A+C}{2}\cos\frac{A-C}{2} = 2\sin B$$

$\Rightarrow \;\; \sin A + \sin C = 2\sin B$, *i.e.* $a + c = 2b$.

17. (c) : Let $f(x) = 4x^2 - 4ax + a^2 - 2a + 2$,

vertex of the parabola is $x = \frac{a}{2}$, as $a < 0 \Rightarrow \frac{a}{2} < 0$

For $f(x)$ to be minimum, $f(x) = 0 \Rightarrow x = \frac{a}{2} < 0$

So, for given values $f(x)$ will be minimum at $x = 0$.
$f(0) = a^2 - 2a + 2 = 3 \Rightarrow a^2 - 2a - 1 = 0$

$$\therefore \;\; a = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}, \; a = 1 - \sqrt{2}$$

18. (b) : $\tan\frac{3\pi}{2}[x+t] = \tan\frac{3\pi}{2}[x]$

$$\Rightarrow \quad \frac{3\pi}{2}[x+t] = n\pi + \frac{3\pi}{2}[x] \Rightarrow 3([x+t] - [x]) = 2n$$

Clearly $n = 3$, $\therefore t = 2$
$\cos^2\pi[x + t] = \cos^2\pi[x]$
$\Rightarrow \; \pi[x + t] = n\pi + \pi[x] \; ; \; [x + t] - [x] = n$
$\therefore \;\; n = 1 \; \Rightarrow t = 1$ and $e^{\{x/3\}}$ is periodic with period 3.
$\therefore \;\;$ LCM of 1, 2, 3 is 6.

19. (c) : $x = \left|\frac{(\hat{i} + 2\hat{j} + 2\hat{k}).(-\hat{i} + 2\hat{j} - \hat{k})}{\sqrt{1+4+1}}\right| = \frac{1}{\sqrt{6}}$

Now, $\cos^{-1}\cos\sqrt{6}x = \cos^{-1}\cos 1 = 1$

20. (d) : $y=\frac{\log x}{x} \Rightarrow \frac{dy}{dx}=-\frac{1}{x^2}\log x+\frac{1}{x}\cdot\frac{1}{x}$

Now, $\frac{dy}{dx}=0 \;\Rightarrow \frac{1}{x^2}(1-\log x)=0$

$\Rightarrow\; \log x=1 \Rightarrow x=e, x<e \;\Rightarrow\; \log x<1,\; x>e \;\Rightarrow\; \log x>1$

$\therefore$ At $x=e$, $\frac{dy}{dx}$ changes sign from positive to negative and hence, y has maximum value at $x = e$ and its value is $\frac{\log e}{e}=e^{-1}$.

21. (a) : $x = 0$ gives the maximum value.

22. (d) : Let the sides a, b, c be $\lambda, \sqrt{3}\lambda, 2\lambda$ respectively.

$c^2=a^2+b^2 \quad \therefore \quad \angle C=\frac{\pi}{2}$

$$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{\lambda^2(3+4-1)}{\lambda^2 4\sqrt{3}}=\frac{\sqrt{3}}{2}$$

$\therefore\quad A=\frac{\pi}{6},\; B=\frac{\pi}{2}-A=\frac{\pi}{3} \quad \therefore\; A:B:C=1:2:3$

23. (a, b) : If $\angle B + \angle D > 180° \Rightarrow B$ and D lie inside the circle through other three points.

Let $\angle B + \angle D < 180°$, then B and D lie outside the circle through other three points.

24. (a, c) : Since, $\sqrt{(a_1+a_2+a_3)(b_1+b_2+b_3)}$
$=\sqrt{a_1b_1}+\sqrt{a_2b_2}+\sqrt{a_3b_3}$

Let $a_1=A_1^2, a_2=A_2^2, a_3=A_3^2$

and $b_1=B_1^2, b_2=B_2^2, b_3=B_3^2$

$\Rightarrow (A_1^2+A_2^2+A_3^2)(B_1^2+B_2^2+B_3^2)=(A_1B_1+A_2B_2+A_3B_3)^2$

$\Rightarrow\; (A_1B_2-A_2B_1)^2+(A_2B_3-A_3B_2)^2+(A_3B_1-A_1B_3)^2=0$

$\Rightarrow\; \frac{A_1}{B_1}=\frac{A_2}{B_2}=\frac{A_3}{B_3}=\lambda$

or $\frac{\sqrt{a_1}}{\sqrt{b_1}}=\frac{\sqrt{a_2}}{\sqrt{b_2}}=\frac{\sqrt{a_3}}{\sqrt{b_3}} \;\therefore\; \frac{a_1}{b_1}=\frac{a_2}{b_2}=\frac{a_3}{b_3}$

25. (a, b, d)

26. (a, b, c, d) : $f(x+1)=\frac{f(x)-5}{f(x)-3}$ ....(i)

$f(x)\cdot f(x+1)-3f(x+1)=f(x)-5$

$\Rightarrow\; f(x)=\frac{3f(x+1)-5}{f(x+1)-1}$

Replacing x by $(x - 1)$, we have

$f(x-1)=\frac{3f(x)-5}{f(x)-1}$ ...(ii)

Using (i), we have

$$f(x+2)=\frac{f(x+1)-5}{f(x+1)-3}=\frac{\frac{f(x)-5}{f(x)-3}-5}{\frac{f(x)-5}{f(x)-3}-3}=\frac{2f(x)-5}{f(x)-2} \quad ...(iii)$$

Using (ii), we have

$$f(x-2)=\frac{3f(x-1)-5}{f(x-1)-1}=\frac{3\left(\frac{3f(x)-5}{f(x)-1}\right)-5}{\frac{3f(x)-5}{f(x)-1}-1}=\frac{2f(x)-5}{f(x)-2} \quad ...(iv)$$

Using (iii) and (iv), we have

$f(x + 2) = f(x - 2) \;\Rightarrow\; f(x + 4) = f(x)$

$\Rightarrow\; f(x)$ is periodic with period 4.

27. (c, d) : Let $f(x) = \sec x + \operatorname{cosec} x$ for $0 < x < \pi, x \neq \pi/2$.

$f'(x) = \sec x \tan x - \operatorname{cosec} x \cot x$
$= \sec^2 x \operatorname{cosec}^2 x(\sin^3 x - \cos^3 x)$

$\Rightarrow\; f'(x)<0$ for $0<x<\frac{\pi}{4}$ and

$f'(x)>0$ for $\frac{\pi}{4}<x<\frac{\pi}{2}$ and $\frac{\pi}{2}<x<\pi$

$\Rightarrow f(x)$ decreases strictly from ∞ to $f\left(\frac{\pi}{4}\right)$ for $0<x<\frac{\pi}{4}$ and increases from $f\left(\frac{\pi}{4}\right)$ to ∞ for $\frac{\pi}{4}<x<\frac{\pi}{2}$.

Also it increases strictly from $-\infty$ to ∞ for $\frac{\pi}{2}<x<\pi$.

Hence for any k the equation $f(x) = k$ has exactly one solution in $\left(\frac{\pi}{2}, \pi\right)$.

If $f\left(\frac{\pi}{4}\right)<k<\infty$, then the equation $f(x) = k$ has exactly one solution in each of the intervals $\left(0, \frac{\pi}{4}\right)$ and $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. If $k<f\left(\frac{\pi}{4}\right)$, then $f(x) = k$ does not have any solution in $\left(0, \frac{\pi}{2}\right)$.

$\therefore f(x) = k$ has 3 solutions only if

$k>f\left(\frac{\pi}{4}\right) \;\Rightarrow\; k>2\sqrt{2}$

(28 – 30) : 28. (d) 29. (c) 30. (c)

Let $\vec{r}=x\hat{i}+y\hat{j}$, $x^2 + y^2 + 8x - 10y + 40 = 0$,

centre $C(-4, 5)$, radius = 1, $p_1 = \max\{(x + 2)^2 + (y - 3)^2\}$

$p_2 = \min\{(x + 2)^2 + (y - 3)^2\}$

Let $P \equiv (-2, 3)$ and M be any point on circle.

$CP=2\sqrt{2},\; CM=1$

$$p_1=(CP+CM)^2=(2\sqrt{2}+1)^2,$$
$$p_2=(CP-CM)^2=(2\sqrt{2}-1)^2,$$
$p_1 + p_2 = 18$

Now, slope of $AB=\left(\dfrac{dy}{dx}\right)_{(2,\,2)}=-2$

Equation of AB is $2x + y = 6$

$$\overrightarrow{OA}=2\hat{i}+2\hat{j},\ \overrightarrow{OB}=3\hat{i},\ \overrightarrow{AB}=\hat{i}-2\hat{j},$$
$$\overrightarrow{AB}\cdot\overrightarrow{OB}=(\hat{i}-2\hat{j})\,(3\hat{i})=3$$

31. (A) → (2), (B) → (1), (C) → (4), (D) → (3)

(A) $y=\tan^{-1}\dfrac{1}{2}+\tan^{-1}b,\quad (0<b<1)$

$$0<\tan^{-1}\left(\frac{1+2b}{2-b}\right)\le\frac{\pi}{4}\ \Rightarrow\ 0<\left(\frac{1+2b}{2-b}\right)\le 1$$
$$\Rightarrow\ 0<(1+2b)\le(2-b)$$
$$\Rightarrow\ 3b\le 1\ \Rightarrow\ b\le\frac{1}{3}\Rightarrow b_{\max}=\frac{1}{3}$$

(B) $\sin^4x + \cos^3x \ge 1$ ...(i)

Since $\sin^2x + \cos^2x = 1$ and $-1 \le \sin x, \cos x \le 1$

(i) can not be > 1

$\therefore\ \sin^4x + \cos^3x = \pm 1$ ...(ii)

(ii) is possible if either, $\sin x = 1$ and $\cos x = 0$ or $\sin x = 0$ and $\cos x = 1$

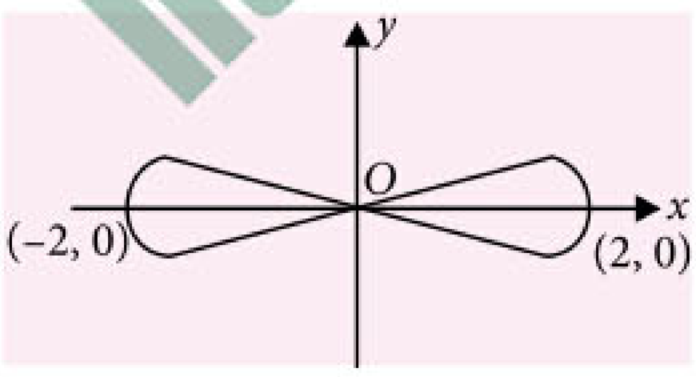
$$\Rightarrow\ x=(4n+1)\frac{\pi}{2},\ x=(2n+1)\frac{\pi}{2}\ \text{ or }\ x=n\pi,\ x=2n\pi$$

$$\text{In}\,(0,\,2\pi),\ x=\frac{\pi}{2},\frac{3\pi}{2}\ \text{ or }\ x=\pi$$

$\therefore$ Number of solutions will be 3.

(C) $4y^2 = x^2(4 - x^2)$ ...(1)

$$\Rightarrow\ y=\frac{1}{2}\sqrt{x^2(4-x^2)}\ \Rightarrow y=\frac{x}{2}\sqrt{(4-x^2)}$$
$$\therefore\ \text{Area}\,(A)=4\times\int_0^2\frac{2x}{2\times 2}\sqrt{(4-x^2)}dx$$



Put $(4-x^2)=t\ \Rightarrow\ -2xdx=dt$

$$\therefore\ A=\frac{-4}{4}\int_4^0\sqrt{t}dt=\int_0^4\sqrt{t}dt=\left[\frac{t^{3/2}}{3/2}\right]_0^4=\frac{2}{3}\times[\sqrt{64}-0]$$
$$\Rightarrow\ A=\frac{16}{3}\ \text{sq. units.}$$

(D) We have, $(1+x^2)\dfrac{dy}{dx}+2xy=1\Rightarrow\dfrac{d}{dx}[(1+x^2)y]=1$

$\Rightarrow\ (1 + x^2)y = x + c$, Since $y(0) = 0$

$\Rightarrow\ (1 + 0)0 = 0 + c\ \Rightarrow\ c = 0$

$$\therefore\ (1+x^2)y=x\Rightarrow y=\frac{x}{(1+x^2)}\Rightarrow y(1)=\frac{1}{1+1}=\frac{1}{2}$$

32. (A) → (2, 4), (B) → (3), (C) → (2), (D) → (5)

(A) The equation is $y=px+\dfrac{1}{p}$, where $p=\dfrac{dy}{dx}\ne 0$,

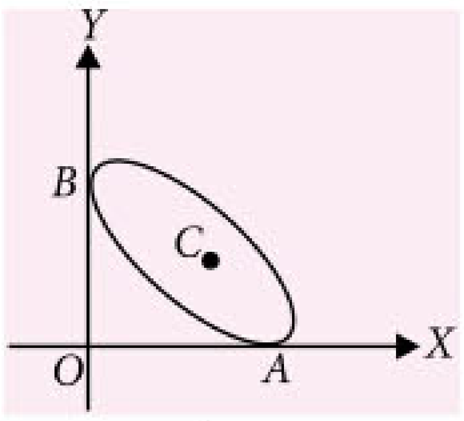
Differentiating w.r.t. x, we get

$$p=p+x\frac{dp}{dx}-\frac{1}{p^2}\frac{dp}{dx}\ \Rightarrow\ p^2=\frac{1}{x}\ \text{ or }\ \frac{dp}{dx}=0$$

Now $\dfrac{dp}{dx}=0\ \Rightarrow\ p = \text{constant} = c$

$\therefore$ Equation is $y=cx+\dfrac{1}{c}$, which is a straight line.

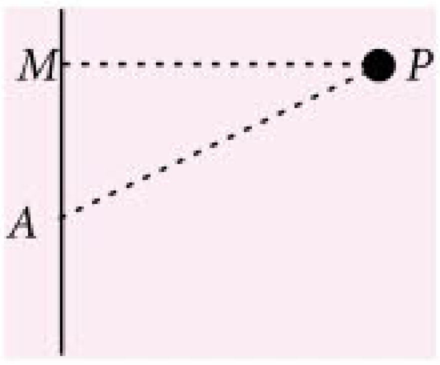
If $p^2=\dfrac{1}{x}$, then $y^2=\dfrac{1}{x}\cdot x^2+x+2x=4x$, which is a parabola.



(B) OA and OB are perpendicular tangents so O lies on the director circle and thus $OC^2 = a^2 + b^2$, where $2a$ is major axis and $2b$ the minor axis. Clearly, C lies on a circle.

(C) Given, $|ay-bx|=c\sqrt{(x-a)^2+(y-b)^2}$

$$\Rightarrow\ \frac{|ay-bx|}{\sqrt{a^2+b^2}}=\frac{c}{\sqrt{a^2+b^2}}\sqrt{(x-a)^2+(y-b)^2}$$



or $PM = kPA$, where PM is the length of perpendicular from $P(x, y)$ on the line $ay-bx=0$ and PA is the length of line segment joining P to the point $A(a, b)$ and A lies on the line, so the locus of P is a straight line through A.

(D) $hxy+ax+by+c=0\Rightarrow\ hx\left(y+\dfrac{a}{h}\right)+b\left(y+\dfrac{c}{b}\right)=0$

Above can be converted into two factors. If $\dfrac{a}{h}=\dfrac{c}{b}$, then it will represent a pair of straight lines otherwise a rectangular hyperbola.

33. (A) → (3), (B) → (2), (C) → (1), (D) → (1)

(A) $\vec{a}, \vec{b}$ are unit vectors.

$$(\vec{a}+2\vec{b})\cdot(5\vec{a}-4\vec{b})=0\ \Rightarrow\ 5|\vec{a}|^2+6\cdot\vec{a}\cdot\vec{b}-8|\vec{b}|^2=0$$
$$\Rightarrow\ 6\vec{a}\cdot\vec{b}=3\ \Rightarrow\ \vec{a}\cdot\vec{b}=\frac{1}{2}\ \Rightarrow\ 2(\vec{a}\cdot\vec{b})=1$$

(B) Let

$\vec{a}=(1,0,3), \vec{b}=(-1,3,4), \vec{c}=(1,2,1)$ and $\vec{d}=(k,2,5)$

a, b, c, d are coplanar if $[\vec{d}\,\vec{b}\,\vec{c}]+[\vec{d}\,\vec{c}\,\vec{a}]+[\vec{d}\,\vec{a}\,\vec{b}]=[\vec{a}\,\vec{b}\,\vec{c}]$

$$\Rightarrow \begin{vmatrix} k & 2 & 5 \\ -1 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix}+\begin{vmatrix} k & 2 & 5 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{vmatrix}+\begin{vmatrix} k & 2 & 5 \\ 1 & 0 & 3 \\ -1 & 3 & 4 \end{vmatrix}=\begin{vmatrix} 1 & 0 & 3 \\ -1 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix}$$

$\Rightarrow\ k=-1$

(C) $(1,1,m), (1,1,m+1), (1,-1,m)$ are coplanar if

$$\begin{vmatrix} 1 & 1 & m \\ 1 & 1 & m+1 \\ 1 & -1 & m \end{vmatrix}=0 \Rightarrow \begin{vmatrix} 1 & 0 & m \\ 1 & 0 & m+1 \\ 1 & -2 & m \end{vmatrix}=0$$

$\Rightarrow 2=0$ which is absurd.

$\therefore$ There is no value of m for which the vectors are coplanar.

(D) $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}$,

$\vec{b}\times(\vec{c}\times\vec{a})=(\vec{b}.\vec{a})\vec{c}-(\vec{c}.\vec{b})\vec{a}$ and

$\vec{c}\times(\vec{a}\times\vec{b})=(\vec{c}.\vec{b})\vec{a}-(\vec{c}.\vec{a})\vec{b}$

Since dot product is commutative, so we have, the sum = 0

34. (A) → (2), (B) → (3), (C) → (1), (D) → (4)

(A) $3\cdot1-2(-2)+5(\lambda)=0 \Rightarrow \lambda=-\frac{7}{5}$

(B) Point $(3, \lambda, \mu)$ lies on $2x+y+z-3=x-2y+z-1$

$\Rightarrow\ 3\cdot2+\lambda+\mu-3=0$ and $3-2\lambda+\mu-1=0$

$\Rightarrow\ \lambda+\mu+3=0$ and $2\lambda-\mu-2=0$ so $\lambda+\mu=-3$

(C) $\sin\theta=\frac{1\cdot4+1(-3)+1\cdot5}{\sqrt{1^2+1^2+1^2}\sqrt{16+9+25}}=\frac{6}{\sqrt{3}\sqrt{50}}$

$\Rightarrow\theta=\sin^{-1}\sqrt{\frac{6}{25}}$

(D) $\cos\theta=\frac{1\cdot3+1(-4)+1\cdot5}{\sqrt{3}\sqrt{16+9+25}}=\frac{4}{\sqrt{3}\sqrt{50}} \Rightarrow \theta=\cos^{-1}\sqrt{\frac{8}{75}}$

35. (A) → (5), (B) → (2), (C) → (4), (D) → (1)

(A) Required number of arrangements $=\frac{6!}{2!\,3!}-20$
$=60-20=40$

(B) Coeff. of $\frac{T_{r+1}}{T_r}=\frac{{}^nC_r}{{}^nC_{r-1}}=\frac{n-r+1}{r}$ put $r=1, 2, \ldots$

$\frac{C_1}{C_0}=\frac{n}{1}, \frac{C_2}{C_1}=\frac{n-1}{2}, \ldots$

Now, $\left(1+\frac{C_1}{C_0}\right)\ldots\left(1+\frac{C_n}{C_{n-1}}\right)=\left(1+\frac{n}{1}\right)\left(1+\frac{n-1}{2}\right)\ldots n$ factors

$=\frac{n+1}{1}\,\frac{n+1}{2}\ldots n$ factors $=\frac{(n+1)^n}{n!}$

(C) Applying $R_1+R_3-2R_2$, we get

$$\Delta=\begin{vmatrix} 0 & 0 & c+e-2d \\ x+2 & x+3 & x+d \\ x+3 & x+4 & x+e \end{vmatrix}$$

Given c, d, e are in A.P. $\therefore\ 2d=c+e$

$$\therefore \begin{vmatrix} 0 & 0 & 0 \\ x+2 & x+3 & x+d \\ x+3 & x+4 & x+e \end{vmatrix}=0$$

(D) $c^2+d^2\ge2cd,\ d^2+e^2\ge2de,\ c^2+e^2\ge2ce$ [∵ A.M. ≥ G.M.]

$\frac{c^2+d^2+e^2}{cd+de+ce}\ge1$.

Also in ΔCDE, $d^2+e^2-c^2=2de\cos C<2de$,

$c^2+d^2-e^2=2cd\cos E<2cd$,

$c^2+e^2-d^2=2ce\cos D<2ce$

Adding, we get

$c^2+d^2+e^2<2(ce+cd+de) \Rightarrow \frac{c^2+d^2+e^2}{ce+de+cd}<2$

36. (4) : $f(3)=3f(1)=3, f(4)=f(2+1+1)$
$=2+1+1=4$ and so on.

In general, we get $f(r)=r$ for $r\in N$.

$$\Rightarrow \lim_{n\to\infty}\frac{\sum_{r=1}^{n}(4r)f(3r)}{n^3}=\lim_{n\to\infty}\frac{12n(n+1)(2n+1)}{6n^3}=4$$

37. (2) : Put $x=r\cos\theta,\ y=r\sin\theta$.

$\therefore\ r^2\sin\theta\cos\theta=1-r^2 \Rightarrow r^2=\frac{2}{2+\sin2\theta},\ \frac{2}{3}\le r^2\le2$

Let $k=x^3y+xy^3=r^2(1-r^2)$ and $r^2=t, \frac{2}{3}\le t\le2$

So, $k=(1-t)\,t$

$\Rightarrow\ \frac{dk}{dt}=1-2t<0$ for $\frac{2}{3}\le t\le2$

So, k is min when t is max *i.e.* for $t=2$, k is min.

When $t=2$, min k is -2.

$\therefore\ x^3y+xy^3+4\ge2$

38. (3) : $\frac{(x-4)^2}{16}+\frac{(y-3)^2}{9}=1$ and point P on the curve $4(1+\cos\theta), 3(1+\sin\theta)$.

The reflection of the point P about the line $x-y-2=0$ can be found using

$$\frac{h-4(1+\cos\theta)}{1}=\frac{k-3(1+\sin\theta)}{-1}$$

$$=\frac{-2(4(1+\cos\theta)-3(1+\sin\theta)-2)}{2}=-(4\cos\theta-3\sin\theta)+1$$

$h = 4(1 + \cos\theta) - 4\cos\theta + 3\sin\theta + 1 = 4 + 3\sin\theta + 1$

and $k = 3(1 + \sin\theta) + 4\cos\theta - 3\sin\theta - 1 = 3 + 4\cos\theta - 1$

$$\left(\frac{h-5}{3}\right)^2 + \left(\frac{k-2}{4}\right)^2 = 1$$

$\therefore$ Equation of image is $16(x - 5)^2 + 9(y - 2)^2 = 144$

$\Rightarrow 16x^2 + 9y^2 - 160x - 36y + 292 = 0$

$\therefore k_1 = -160,\ k_2 = 292 \Rightarrow \frac{k_1 + k_2}{44} = \frac{132}{44} = 3$

39. (2) : $5x\left(1 + \frac{1}{x^2 + y^2}\right) = 12 \Rightarrow x = \frac{12}{5}\frac{1}{1 + \frac{1}{x^2 + y^2}}$ ... (i)

Also $5y\left(1 - \frac{1}{x^2 + y^2}\right) = 4 \Rightarrow y = \frac{4}{5}\frac{1}{1 - \frac{1}{x^2 + y^2}}$ ...(ii)

Squaring and adding (i) and (ii), we have

$$x^2 + y^2 = \left(\frac{4}{5}\right)^2 \left[\frac{9}{\left(1 + \frac{1}{x^2 + y^2}\right)^2} + \frac{1}{\left(1 - \frac{1}{x^2 + y^2}\right)^2}\right]$$

$\Rightarrow \frac{t}{t^2} = \left(\frac{4}{5}\right)^2 \left\{\frac{9}{(t+1)^2} + \frac{1}{(t-1)^2}\right\}$, where $x^2 + y^2 = t$

$\Rightarrow \frac{1}{t} = \left(\frac{4}{5}\right)^2 \left\{\frac{10t^2 - 16t + 10}{(t^2 - 1)^2}\right\}$

$\Rightarrow 25(t^4 - 2t^2 + 1) = 16t(10t^2 - 16t + 10)$

$\Rightarrow 25t^2\left(t^2 + \frac{1}{t^2} - 2\right) = 16(2t^2)\left\{5t + \frac{5}{t} - 8\right\}$

$\Rightarrow 25t^2\left(\left(t + \frac{1}{t}\right)^2 - 4\right) = 32t^2\left\{5\left(t + \frac{1}{t}\right) - 8\right\}$

$\Rightarrow 25\{z^2 - 4\} = 32\{5z - 8\}$, where $t + \frac{1}{t} = z$

$\Rightarrow (5z - 6)(5z - 26) = 0 \quad \therefore \quad z = \frac{6}{5}$ or $z = \frac{26}{5}$

$\Rightarrow t + \frac{1}{t} = \frac{6}{5}$ or $t + \frac{1}{t} = 5 + \frac{1}{5}$

$\because t + \frac{1}{t} \geq 2$ so, either $t = 5$ or $\frac{1}{5}$

Hence, $x^2 + y^2 = 5$ or $x^2 + y^2 = \frac{1}{5}$

$\therefore x = \frac{12}{5}\frac{1}{\left(1 + \frac{1}{5}\right)}$ or $x = \frac{12}{5}\left(\frac{1}{1+5}\right)$

$\Rightarrow x = \frac{12}{5} \cdot \frac{5}{6} = 2$ or $x = \frac{12}{5} \cdot \frac{1}{6} = \frac{2}{5}$

$\Rightarrow y = 1$ or $y = -\frac{1}{5}$

Hence, $(x, y) = (2, 1)$ or $\left(\frac{2}{5}, -\frac{1}{5}\right)$

Hence, the given system possesses two solutions.

40. (1) : $x - y = \frac{y - z}{yz},\ y - z = \frac{z - x}{zx},\ z - x = \frac{x - y}{xy}$

Multiplying above all, we get $xyz = 1$ (As $xyz > 0$)

◆

Type of Academic Programs Offered at IITs on the basis of JEE Advanced

Programs	Full Form	Minimum Duration
B. Tech.	Bachelor of Technology	4 years
B.S.	Bachelor of Science	4 years
B. Arch.	Bachelor of Architecture	5 years
Dual Degree B. Tech.- M.Tech.	Dual Degree Bachelor of Technology and Master of Technology	5 years
Dual Degree B.S. - M.S.	Dual Degree Bachelor of Science and Master of Science	5 years
Integrated M. Tech.	Integrated Master of Technology	5 years
Integrated M. Sc.	Integrated Master of Science	5 years

For more information, refer to latest prospectus of JEE Advanced.

Note: Only those academic programs for which admission is based on JEE (Advanced) examination are shown here. These Institutes also have other academic programs, viz., B.Des., M. Tech., M.Sc., M.Des., Ph. D., etc. with different admission criteria.

MOCK TEST PAPER

ISI

Indian Statistical Institute

Exam on
13th May
2018

*ALOK KUMAR, B.Tech, IIT Kanpur

PART-A

1. Let a, b and c be real numbers such that $a + 2b + c = 4$ then the maximum value of $ab + bc + ca$ is

(a) 1 (b) 2 (c) 3 (d) 4

2. For the equation $3x^2 + px + 3 = 0$, $p > 0$, if one root is the square of the other then value of p is

(a) 1/3 (b) 1 (c) 3 (d) 2/3

3. If α and β are the roots of the equation $x^2 - x + 1 = 0$ then $\alpha^{2009} + \beta^{2009} =$

(a) 1 (b) 2 (c) –1 (d) –2

4. If α, β, γ are the roots of the equation $x^3 + px + q = 0$, then the value of the determinant $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$ is

(a) 4 (b) 2 (c) 0 (d) –2

5. The number of points (p, q) such that $p, q \in \{1, 2, 3, 4\}$ and the equation $px^2 + qx + 1 = 0$ has real roots is

(a) 7 (b) 8
(c) 9 (d) none of these

6. A and B are two non singular matrices so that $A^6 = I$ and $AB^2 = BA$ $(B \neq I)$. A value of K so that $B^K = I$ is

(a) 31 (b) 32 (c) 63 (d) 64

7. For each real number x such that $-1 < x < 1$, let $A(x)$ be the matrix $(1-x)^{-1}\begin{bmatrix} 1 & -x \\ -x & 1 \end{bmatrix}$ and $z = \dfrac{x+y}{1+xy}$. Then,

(a) $A(z) = A(x) + A(y)$ (b) $A(z) = A(x)\,[A(y)]^{-1}$
(c) $A(z) = A(x)A(y)$ (d) $A(z) = A(x) - A(y)$

8. If A and B are symmetric matrices of same order and $X = AB + BA$ and $Y = AB - BA$, then $(XY)^T$ is equal to

(a) XY (b) YX
(c) $-YX$ (d) none of these

9. A square matrix A is said to be nilpotent of index m. If $A^m = 0$, now, if for this A $(I - A)^n = I + A + A^2 + \ldots + A^{m-1}$, then n is equal to

(a) 0 (b) m (c) $-m$ (d) –1

10. If $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, then A^8 equals

(a) $4B$ (b) $128\,B$
(c) $-128\,B$ (d) $-64\,B$

11. M is the mid point of side AB of equilateral triangle ABC. P is a point on BC such that $AP + PM$ is minimum. If $AB = 20$, then $AP + PM$ is

(a) $10\sqrt{7}$ (b) $10\sqrt{3}$ (c) $10\sqrt{5}$ (d) 10

12. The reflection of the curve $xy = 1$ in the line $y = 2x$ is the curve $12x^2 + rxy + sy^2 + t = 0$, then the value of '$r$' is

(a) –7 (b) 25 (c) –175 (d) 90

13. Let S be the set of all values of α such that the points $(\alpha, 6)$, $(-5, 0)$ and $(5, 0)$ form an isosceles triangle. Then the value of $\sum\limits_{\alpha \in S} \alpha^2$ is

(a) 356 (b) 18 (c) 178 (d) 338

14. The line $L_1 \equiv 4x + 3y - 12 = 0$ intersects the x-axis and y-axis at A and B respectively. A variable line perpendicular to L_1 intersect the x and y-axes at P and Q respectively. The locus of the circumcentre of triangle ABQ is

(a) $6x - 8y + 7 = 0$ (b) $6x + 8y - 25 = 0$
(c) $8x - 6y + 7 = 0$ (d) $14x - 12y + 3 = 0$

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15. Triangle is formed by the lines $x + y = 0$, $x - y = 0$ and $lx + my = 1$. If l and m vary subject to the condition $l^2 + m^2 = 1$, then the locus of its circumcentre is
(a) $(x^2 - y^2)^2 = x^2 + y^2$
(b) $(x^2 + y^2)^2 = (x^2 - y^2)$
(c) $(x^2 + y^2)^2 = 4x^2y^2$
(d) $(x^2 - y^2)^2 = (x^2 + y^2)^2$

16. The domain of the function

$$f(x)=\frac{1}{x}+\sin^{-1}x+\frac{1}{\sqrt{x-2}}\text{ is}$$

(a) $[-1, 1]\backslash\{0\}$ (b) $[-1, 1]$
(c) $(-1, 0)$ (d) ϕ

17. Let $f:R\to\left[0,\frac{\pi}{2}\right)$ be defined by $f(x) = \tan^{-1}(x^2 + x + a)$. Then the set of values of a for which f is onto is
(a) $[0, \infty)$ (b) $\left[\frac{1}{4},\infty\right)$
(c) $\left(-\infty,\frac{1}{4}\right]$ (d) $\left\{\frac{1}{4}\right\}$

18. The domain of the function

$$f(x)=[9^x+27^{2/3(x-2)}-219-3^{2(x-1)}]^{1/4}$$

(a) $[-3, 3]$ (b) $[3, \infty)$ (c) $\left[\frac{5}{2},\infty\right)$ (d) $[0, 1]$

19. Domain of the function

$$f(x)=\log_2\left(\log_4\left(\log_2\left(\log_3(x^2+4x-23)\right)\right)\right)\text{ is}$$

(a) $(-8, 4)$ (b) $(-\infty, -8) \cup (4, \infty)$
(c) $(-4, 8)$ (d) $(-\infty, -4) \cup (8, \infty)$

20. Range of the function $y=\dfrac{2^x-2^{-x}}{2^x+2^{-x}}$ is
(a) R (b) $(-1, 1)$
(c) $[-1, 1]$ (d) $(0, 1)$

21. If a_n and b_n are positive integers and $a_n+\sqrt{2}b_n=(2+\sqrt{2})^n$, then $\lim\limits_{n\to\infty}\left(\dfrac{a_n}{b_n}\right)=$
(a) $\sqrt{2}$ (b) 2 (c) $e^{\sqrt{2}}$ (d) e^2

22. If $f(0) = 0$ and that 'f' is differentiable at $x = 0$, and 'k' is a positive integer. Then

$$\lim_{x\to 0}\frac{1}{x}\left[f(x)+f\left(\frac{x}{2}\right)+f\left(\frac{x}{3}\right)+......+f\left(\frac{x}{k}\right)\right]$$

(a) $k\cdot f'(0)$ (b) $\left(\sum_{r=1}^{K}\frac{1}{r}\right)f'(0)$
(c) $\sum_{r=1}^{K}\frac{1}{r}$ (d) does not exist

23. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{(1-\sin x)(8x^3-\pi^3)\cos x}{(\pi-2x)^4}$

(a) $-\frac{\pi^2}{16}$ (b) $\frac{3\pi^2}{16}$ (c) $\frac{\pi^2}{16}$ (d) $-\frac{3\pi^2}{16}$

24. Let $f: R^+ \to R^+$ be a function satisfying the relation $f(x \cdot f(y)) = f(xy) + x$ for all $x, y \in R^+$. Then $\lim\limits_{x\to 0}\left(\dfrac{(f(x))^{1/3}-1}{(f(x))^{1/2}-1}\right)=$
(a) 1 (b) 1/2 (c) 2/3 (d) 3/2

25. $\lim\limits_{n\to\infty}\sum_{r=1}^{n}\cot^{-1}\left(r^2+\frac{3}{4}\right)=$
(a) $\tan^{-1}(2)$ (b) $\pi/4$
(c) $\pi/2$ (d) $\tan^{-1}(3)$

PART-B

1. A variable line passing through a fixed point P cuts the given parabola $y^2 = 4ax$ at Q and R. Now a variable point S is chosen on this line in such a way that PQ, PS and PR are in H.P. Prove that the locus of S is a straight line whose slope is independent of the abscissa of the point P.

2. In any scalene triangle ABC the line joining the orthocentre to the circumcentre is parallel to the side AC, then prove that $\tan A \tan C = 3$.

3. Solve the equation $\sin^{2n-1}x + 2\cos^{2n-1}x = 2$, where $n \in N$.

4. Find the order of the differential equation of the family of ellipse having fixed centre and given eccentricity.

5. Show that every skew-symmetric matrix of odd order is singular.

6. A and B participate in a tournament of "best of 7 games". It is equally likely that either A wins or B wins or the game ends in a draw. What is the probability that A wins the tournament.

7. Check the differentiability of the function $f(x) = \max\{\sin^{-1}|\sin x|, \cos^{-1}|\sin x|\}$.

8. In an acute angled triangle ABC, 'O' is a point inside it, such that $\angle AOB = \angle AOC = \angle BOC$. Then show that the minimum value of the sum of the distances of 'O' from the vertices is $\sqrt{6(r\Delta)^{2/3}+2\sqrt{3}\Delta}$.

9. Let $f(x) = 1 + 6x - x^2$, $\forall\ x \in R$ and $g(x)=\begin{cases}\max f(t); & x\le t\le x+1, \quad 0\le x<4\\ \min(x+3); & \qquad\qquad\quad 4\le x\le 6\end{cases}$ Discuss the continuity and differentiability of $g(x)$ in $(0, 6)$.

10. If $\sqrt{2}\cos A=\cos B+\cos^3 B$, $\sqrt{2}\sin A=\sin B-\sin^3 B$, prove that $\sin(A-B)=\pm\frac{1}{3}$.

11. Find the values of 'a' if there exists a tangent to the ellipse $a^2x^2+\frac{y^2}{2}=1$ such that chord intercepted between given tangent and hyperbola $\frac{x^2}{a^2}-2y^2=1$ subtends a right angle at the centre of the curves.

12 (a) Show that $\left(\frac{ai-1}{ai+1}\right)^n\cdot e^{2ni\tan^{-1}a}=(-1)^n$.

(b) If p, q, r are real numbers and z is a complex number such that $p^2 + q^2 + r^2 = a^2$ and $q + ir = (a + p)z$. Prove that $\frac{p+iq}{a+r}=\frac{1+iz}{1-iz}$ and $\frac{r+ip}{a+q}=\frac{i(1-z)}{1+z}$.

13. If $A(\vec{a}), B(\vec{a}+\vec{b}), C(\vec{b}), D(\lambda\vec{a})$ and $E(\lambda\vec{b})$ be the vertices of a regular pentagon $ABCDE$ then find λ. Hence prove that the area of pentagon is $|\vec{a}+\vec{b}|\left[1+|\lambda|+\frac{\lambda^2}{2}\right]$.

14. Consider the sequence 1, 2, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5..... Find the 2000th term of the sequence. Find also the sum of first 2000 terms.

15. If $(1+x)^n=\sum_{r=0}^{n}{}^nC_r x^r$ and $\sum_{r=0}^{n}\frac{1}{{}^nC_r}=a$. Find the value of $\sum\sum_{0\le i<j\le n}\left(\frac{i}{{}^nC_i}+\frac{j}{{}^nC_j}\right)$ in term of 'a' and 'n'.

SOLUTIONS

PART-A

1. (d) : Let $ab + bc + ca = x$

$\Rightarrow 2b^2 + 2(c - 2)b - 4c + c^2 + x = 0$

Since $b \in R$

$\therefore\ c^2 - 4c + 2x - 4 \le 0$

Since $c \in R$

$\therefore\ x \le 4$

2. (c) : $\alpha+\alpha^2=-\frac{p}{3}=u$

$(\alpha + \alpha^2)^3 = (\alpha^3 + \alpha^6) + 3(\alpha + \alpha^2)$gives

$u^3 = 2 + 3u$

i.e., $u^3 - 3u - 2 = 0 \Rightarrow u = -1\ \therefore\ p = 3$

$\alpha^3 = 1$

3. (a) : $x=\frac{1\pm i\sqrt{3}}{2}$

$\therefore\ \alpha = -\omega,\ \beta = -\omega^2$

4. (c) : Since α, β, γ are the roots of $x^3 + px + q = 0$

$\therefore\ \alpha + \beta + \gamma = 0$

Applying $C_1 \to C_1 + C_2 + C_3$, then

$$\begin{vmatrix}\alpha+\beta+\gamma & \beta & \gamma\\ \alpha+\beta+\gamma & \gamma & \alpha\\ \alpha+\beta+\gamma & \alpha & \beta\end{vmatrix}=\begin{vmatrix}0 & \beta & \gamma\\ 0 & \gamma & \alpha\\ 0 & \alpha & \beta\end{vmatrix}=0$$

5. (a) : $px^2 + qx + 1 = 0$ has real roots if $q^2 - 4p \ge 0$ or $q^2 \ge 4p$

Since $p, q \in \{1, 2, 3, 4\}$

The required points are(1,2), (1,3),(1, 4), (2,3),(2,4), (3,4),(4,4)

So the required number is 7.

6. (c) : $A^5(AB^2) = A^5BA$

$\Rightarrow B^2 = A^5BA$

$\Rightarrow B^4 = (A^5BA)(A^5BA) = A^5B^2A = A^5(A^5BA)A$

$\Rightarrow B^4 = A^4BA^2$

$\Rightarrow B^8 = (A^4BA^2)(A^4BA^2) = A^4B^2A^2 = A^4(A^5BA)A^2$

$\Rightarrow B^8 = A^3BA^3$

$\Rightarrow B^{16} = (A^3BA^3)(A^3BA^3) = A^3B^2A^3 = A^3(A^5BA)A^3$
$= A^2BA^4$

$A^{32} = (A^2BA^4)(A^2BA^4) = A^2B^2A^4 = A^2(A^5BA)A^4 = ABA^5$

$A^{64} = (ABA^5)(ABA^5) = AB^2A^5 = A(A^5BA)A^5 = B$

$\Rightarrow A^{63} = I$

7. (c) : $A(z)=A\left(\frac{x+y}{1+xy}\right)$

$$=\left[\frac{1+xy}{(1-x)(1-y)}\right]\begin{bmatrix}1 & -\left(\frac{x+y}{1+xy}\right)\\ -\left(\frac{x+y}{1+xy}\right) & 1\end{bmatrix}$$

$\therefore\ A(x)\cdot A(y) = A(z)$

8. (c) : $X = AB + BA \Rightarrow X^T = X$

and $Y = AB - BA \Rightarrow Y^T = -Y$

Now, $(XY)^T = Y^T \times X^T = -YX$

9. (d) : Let $B = I + A + A^2 + ... + A^{m-1}$

$\Rightarrow B(I - A) = (I + A + A^2 + ... + A^{m-1})(I - A)$

$= I - A^m = I$

$\Rightarrow B = (I - A)^{-1} \Rightarrow n = -1.$

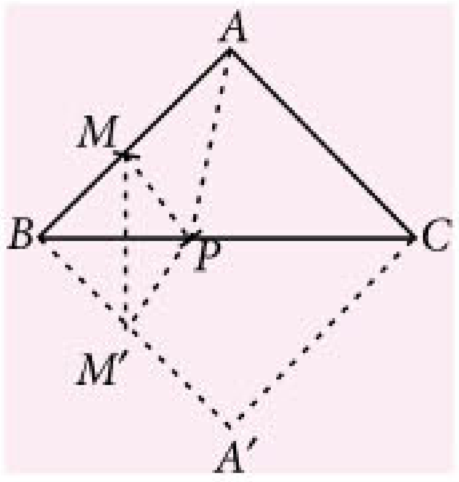
10. (b) : We have $A = iB$

$$\Rightarrow A^2 = (iB)^2 = i^2B^2 = -B^2 = -\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = -2B$$

$\Rightarrow A^4 = (-2B)^2 = 4B^2 = 4(2B) = 8B$

$\Rightarrow (A^4)^2 = (8B)^2 \quad \Rightarrow A^8 = 64B^2 = 128B$

11. (a) : Take the reflection of ΔABC in BC.



$PM = PM'$

$PA + PM = PA + PM'$ it is minimum when M' PA lies in a line.

Now apply cosine rule in triangle ABM'

We will get $AM' = 10\sqrt{7}$

12. (a) : The reflection of (α, β) in the line $y = 2x$ is

$$(\alpha_1, \beta_1) = \left(\frac{4\beta - 3\alpha}{5}, \frac{4\alpha + 3\beta}{5}\right) = \alpha_1\beta_1 = 1$$

$\Rightarrow 12\alpha^2 - 7\alpha\beta - 12\beta^2 + 25 = 0$

13. (a) : α can take 5 values :0,3,−3,13, −13

14. (a) : Clearly circumcentre of triangle ABQ will lie on perpendicular bisector of line AB, which is $6x - 8y + 7 = 0$

15. (a) : Circumcentre of the triangle formed by the given lines is given by

$$\left(\frac{l}{l^2 - m^2}, \frac{m}{l^2 - m^2}\right)$$

Hence the locus of this point is $(x^2 - y^2)^2 = x^2 + y^2$

16. (d) : $x \neq 0, -1 \leq x \leq 1, x - 2 > 0$

17. (d) : $x^2 + x + a = 0$ has a real solution

$\Rightarrow 1 - 4a \geq 0$

18. (c) : We must have

$$9^x + 27^{\frac{2}{3}(x-2)} - 219 - 3^{2(x-1)} \geq 0$$

$$\Rightarrow (3^x)^2 + 3^{2(x-2)} - 219 - 3^{2x-2} \geq 0$$

$$\Rightarrow (3^{2x}) + \frac{3^{2x}}{81} - 219 - \frac{3^{2x}}{9} \geq 0$$

$$\Rightarrow \left(1 + \frac{1}{81} - \frac{1}{9}\right)3^{2x} \geq 219$$

$$\Rightarrow \frac{73}{81} \times 3^{2x} \geq 219$$

$\Rightarrow 3^{2x} \geq 3 \times 81 = 3^5$

$\Rightarrow 2x \geq 5$

$$\Rightarrow x \geq \frac{5}{2} \quad \Rightarrow \text{Domain is} \left[\frac{5}{2}, \infty\right).$$

19. (b) : The given function is defined when

$\log_2 \log_3(x^2 + 4x - 23) > 1$

i.e., when $\log_3(x^2 + 4x - 23) > 2$

i.e., when $x^2 + 4x - 23 > 3^2$

i.e., when $x^2 + 4x - 32 > 0$

i.e., when $x < -8$ or $x > 4$

20. (b) : $2^x + 2^{-x}$ is always > 0 *i.e.*, domain is R

$$y = \frac{2^x - 2^{-x}}{2^x + 2^{-x}} = \frac{2^{2x} - 1}{2^{2x} + 1}$$

$$\Rightarrow \frac{1 + y}{1 - y} = \frac{2.2^{2x}}{2} \text{ (Componendo Dividendo)}$$

$= 2^{2x} > 0$

$$\Rightarrow \frac{1 + y}{1 - y} > 0 \quad i.e., \quad \frac{(1 + y)^2}{1 - y^2} > 0$$

$\Rightarrow 1 - y^2 > 0 \quad \Rightarrow \quad -1 < y < 1$

21. (a) : We have $a_n + \sqrt{2}b_n = (2 + \sqrt{2})^n$

$$\Rightarrow a_n - \sqrt{2}b_n = (2 - \sqrt{2})^n$$

$$\text{Therefore } a_n = \frac{1}{2}\left[(2 + \sqrt{2})^n + (2 - \sqrt{2})^n\right]$$

$$\text{and } b_n = \frac{\left[(2 + \sqrt{2})^n - (2 - \sqrt{2})^n\right]}{2\sqrt{2}}$$

$$\text{Therefore } \frac{a_n}{b_n} = \sqrt{2}\frac{\left[(2 + \sqrt{2})^n + (2 - \sqrt{2})^n\right]}{\left[(2 + \sqrt{2})^n - (2 - \sqrt{2})^n\right]}$$

$$= \sqrt{2}\frac{\left[1 + \left(\frac{2 - \sqrt{2}}{2 + \sqrt{2}}\right)^n\right]}{\left[1 - \left(\frac{2 - \sqrt{2}}{2 + \sqrt{2}}\right)^n\right]}$$

$$\text{Hence } \lim_{n \to \infty}\left(\frac{a_n}{b_n}\right) = \sqrt{2}\left(\frac{1 + 0}{1 - 0}\right)\left(\because \frac{2 - \sqrt{2}}{2 + \sqrt{2}} < 1\right) = \sqrt{2}$$

22. (b) : $$l=\lim_{x\to 0}\left\{\frac{f(x)-f(0)}{x-0}+\frac{f\left(\frac{x}{2}\right)-f(0)}{\frac{x}{2}-0}+\ldots\ldots+\frac{f\left(\frac{x}{k}\right)-f(0)}{\frac{x}{2}-0}\right\}$$

$$=\left(1+\frac{1}{2}+\frac{1}{3}+\ldots\ldots+\frac{1}{k}\right)f'(0).$$

23. (d) : Let $f(x)=\dfrac{(1-\sin x)(8x^3-\pi^3)\cos x}{(\pi-2x)^4}$

$$=\frac{(1-\sin x)\cos x(2x-\pi)(4x^2+2\pi x+\pi^2)}{(2x-\pi)^4}$$

$$=\frac{(1-\sin x)\cos x(4x^2+2\pi x+\pi^2)}{(2x-\pi)^3}$$

Therefore $\lim\limits_{x\to\frac{\pi}{2}} f(x)=\lim\limits_{x\to\frac{\pi}{2}}\dfrac{(1-\sin x)\cos x}{(2x-\pi)^3}.(3\pi^2)$

$$\lim_{x\to\frac{\pi}{2}} f(x)=\lim_{x\to\frac{\pi}{2}}\frac{(1-\sin x)\cos x}{(2x-\pi)^3}(3\pi^2)$$

Put $2x-\pi=y$ so that $y\to 0$ as $x\to\pi/2$. Therefore now

$$\frac{(1-\sin x)\cos x}{(2x-\pi)^3}=\frac{\left[1-\sin\left(\frac{\pi+y}{2}\right)\right]\cos\left(\frac{\pi+y}{2}\right)}{y^3}$$

$$=\frac{\left(1-\cos\frac{y}{2}\right)\left(-\sin\frac{y}{2}\right)}{y^3}=-\left(\frac{2\sin^2\frac{y}{4}}{y^2}\right)\left(\frac{\sin\frac{y}{2}}{y}\right)$$

$$=-2\left(\frac{\sin\frac{y}{4}}{y/4}\right)^2.\frac{1}{16}.\left(\frac{\sin\frac{y}{2}}{y/2}\right).\frac{1}{2}=\frac{-1}{16}\left(\frac{\sin\frac{y}{4}}{y/4}\right)^2\left(\frac{\sin\frac{y}{2}}{y/2}\right)$$

Therefore, $\lim\limits_{x\to\frac{\pi}{2}} f(x)=\dfrac{-3\pi^2}{16}\times 1\times 1$

24. (c) : Given relation is $f(x\cdot f(y))=f(xy)+x$ ... (1)

Interchanging x and y in Eq. (1), we have

$f(y\cdot f(x))=f(yx)+y$

Again replacing x with $f(x)$in Eq. (1), we get

$f(f(x)\cdot f(y))=f(y\cdot f(x))+f(x)$ ...(2)

Therefore, Eqs. 2 only imply

$f(f(x)\cdot f(y))=f(xy)+y+f(x)$

Again interchanging x and y in Eq. (2), we have

$f(f(y)\cdot f(x))=f(yx)+x+f(y)$ ...(3)

Equations (2) and (3) imply

$f(xy)+y+f(x)=f(yx)+x+f(y)$

Suppose $f(x)-x=f(y)-y=\lambda$

Substituting $f(x)=\lambda+x$ in Eq. (1), we have

$x\cdot f(y)+\lambda=(xy+\lambda)+x$

$\Rightarrow\ x\cdot f(y)=xy+x$

Therefore $x(y+\lambda)=xy+x$ $[\because f(y)=\lambda+y]$

$\Rightarrow\ \lambda x=x$

$\Rightarrow\ \lambda=1$ $(\because x>0)$

So $f(x)=x+\lambda=x+1$

Hence $\lim\limits_{x\to 0}\dfrac{(f(x))^{1/3}-1}{(f(x))^{1/2}-1}=\lim\limits_{x\to 0}\dfrac{(1+x)^{1/3}-1}{(1+x)^{1/2}-1}$

$$=\lim_{x\to 0}\left(\frac{(1+x)^{1/3}-1}{1+x-1}\right).\left(\frac{1+x-1}{(1+x)^{1/2}-1}\right)=\frac{1/3}{1/2}=\frac{2}{3}$$

25. (a) : $\cot^{-1}\left(r^2+\dfrac{3}{4}\right)=\tan^{-1}\left(\dfrac{1}{r^2+\frac{3}{4}}\right)$

$$=\tan^{-1}\left(\frac{1}{1+\left(r^2-\frac{1}{4}\right)}\right)=\tan^{-1}\left(\frac{1}{1+\left(r+\frac{1}{2}\right)\left(r-\frac{1}{2}\right)}\right)$$

$$=\tan^{-1}\left(\frac{\left(r+\frac{1}{2}\right)-\left(r-\frac{1}{2}\right)}{1+\left(r^2+\frac{1}{4}\right)}\right)=\tan^{-1}\left(r+\frac{1}{2}\right)-\tan^{-1}\left(r-\frac{1}{2}\right)$$

FUN FACT

***Pizza and Math** : Are they related?*

Let a cylinder of base radius z and height a then

Volume of cylinder = $pi\times z\times z\times a$

which makes *pizza*

PART-B

1. Let $P \equiv (\alpha, \beta)$ and any line through 'P' be

$$\frac{x-\alpha}{\cos\theta} = \frac{y-\beta}{\sin\theta} = r$$

$\Rightarrow$ Any point on this line is $(\alpha + r\cos\theta, \beta + r\sin\theta)$

Points where it meets the parabola $y^2 = 4ax$, can be obtained from ;

$$(\beta + r\sin\theta)^2 = 4a(\alpha + r\cos\theta)$$

$$\Rightarrow r^2\sin^2\theta + 2r(\beta\sin\theta - 2a\cos\theta) + \beta^2 - 4a\alpha = 0$$

$$\Rightarrow PQ + PR = \frac{2(2a\cos\theta - \beta\sin\theta)}{\sin^2\theta}$$

$$\text{and } PQ \cdot PR = \frac{\beta^2 - 4a\alpha}{\sin^2\theta}$$

Since PQ, PS and PR are in H.P.

$$\Rightarrow PS = \frac{(\beta^2 - 4a\alpha)}{(2a\cos\theta - \beta\sin\theta)}$$

Let $S \equiv (h, k)$

$\Rightarrow$ $h = \alpha + PS.\cos\theta$, $k = \beta + PS.\sin\theta$

$\Rightarrow$ $\beta^2 = 4a\alpha = 2a(h - \alpha) - \beta(k - \beta)$

Hence locus of 'S' is $2ax - y\beta + 2a\alpha = 0$,

which is clearly a straight line whose slope is $\frac{2a}{\beta}$, that does not depend upon the abscissa of point 'P'.

2. Here, H-orthocentre, O-circum-centre.

$HD = 2R\cos C \cos A$

$HD = OE = 2R\cos A \cos C$

$OC = R$

$\angle EOC = B$

From ΔEOC,

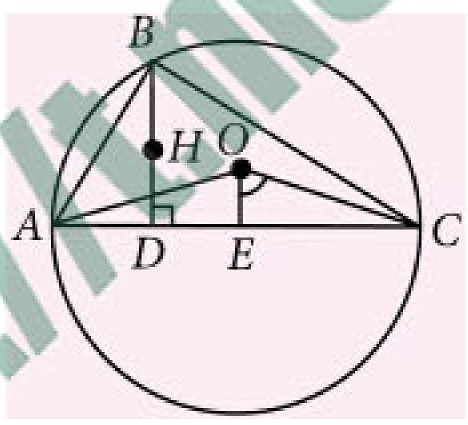
$R\cos B = 2R\cos A\cos C$

$\Rightarrow$ $-\cos(A + C) = 2\cos A \cos C$

$$\Rightarrow \frac{\sin A\sin C - \cos A\cos C}{\cos A\cos C} = 2$$

$\Rightarrow$ $\tan A \tan C - 1 = 2$

$\Rightarrow$ $\tan A \tan C = 3$



3. Obviously no solution is possible if $\frac{\pi}{2} < x < 2\pi$ as LHS < 2.

$0 < x < \frac{\pi}{2}$, then

$$\text{LHS} = \sin^{2n-1}x + 2\cos^{2n-1}x < \sin^2 x + 2\cos^2 x$$

$$= 1 + \cos^2 x < 2 \text{ when } n \in N - \{1\}.$$

Obviously, a solution exists only when $x = 0$

$\Rightarrow$ The general solution is $x = 2m\pi$, $m \in I$.

When $n = 1$, $\sin x + 2\cos x = 2$

$$\sin\frac{x}{2}\left(2\sin\frac{x}{2} - \cos\frac{x}{2}\right) = 0$$

$$\Rightarrow \text{either } x = 2k_1\pi \text{ or } x = 2k_2\pi + 2\tan^{-1}\frac{1}{2}, k_1, k_2 \in I$$

4. Let $\frac{((y-\beta)-m(x-\alpha))^2}{a^2} + \frac{(m(x-\alpha)+(y-\beta))^2}{b^2} = 1$

be the equation of the ellipse having (α, β) as its centre,

where $e^2 = 1 - \frac{b^2}{a^2}$

From both the equations we get two independent parameters

$\Rightarrow$ order of differential equation is 2.

5. Since, $|A'| = (-1)^n |A|$

$\Rightarrow$ $|A|(1 - (-1)^n) = 0$ $[\because |A'| = |A|]$

As n is odd $\Rightarrow$ $|A| = 0$

Hence, A is singular.

6. Let the probability that A wins the tournament $= x$

The probability that B wins the tournament $= x$

The probability that the game ends in a draw $= y$ so that $2x + y = 1$ $\Rightarrow$ $x = \frac{1-y}{2}$.

Probability that the game ends in a draw

Case I : one draw, 3 wins and 3 losses $= \frac{7!}{3!3!}\left(\frac{1}{3}\right)^7$

Case II : three draws, 2 wins and 2 losses $= \frac{7!}{3!2!2!}\left(\frac{1}{3}\right)^7$

Case III : all draws $= \frac{7!}{5!}\left(\frac{1}{3}\right)^7$

$$y = \frac{1}{3^7}\left(\frac{7!}{3!3!} + \frac{7!}{3!2!2!} + \frac{7!}{5!} + 1\right) = \frac{131}{729}$$

$$\Rightarrow x = \frac{1}{2}\left(1 - \frac{131}{729}\right) = \frac{299}{729}.$$

7. $\sin^{-1}|\sin x|$ is periodic with period π

$$\Rightarrow \sin^{-1}|\sin x| = \begin{cases} x & , \quad n\pi \le x \le n\pi + \frac{\pi}{2} \\ \pi - x & , \quad n\pi + \frac{\pi}{2} \le x \le n\pi + \pi \end{cases}$$

$$\text{Also } \cos^{-1}|\sin x| = \frac{\pi}{2} - \sin^{-1}|\sin x|$$

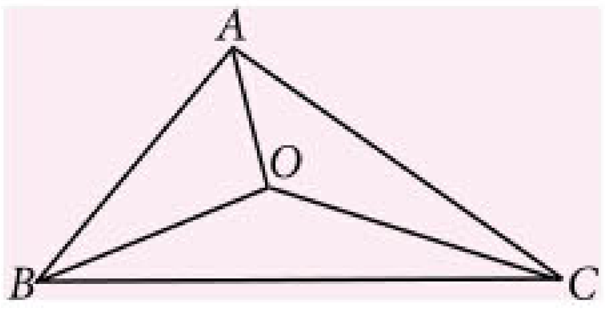
$$\Rightarrow f(x) = \max\begin{cases} x, \frac{\pi}{2} - x & , \quad n\pi \le x \le n\pi + \frac{\pi}{2} \\ \pi - x, x - \frac{\pi}{2} & , \quad n\pi + \frac{\pi}{2} \le x \le n\pi + \pi \end{cases}$$

$$\Rightarrow f(x)=\begin{cases}\frac{\pi}{2}-x, & n\pi\le x\le n\pi+\frac{\pi}{4}\\ x, & n\pi+\frac{\pi}{4}<n\pi+\frac{\pi}{2}\\ \pi-x, & n\pi+\frac{\pi}{2}<x\le n\pi+\frac{3\pi}{4}\\ x-\frac{\pi}{2}, & n\pi+\frac{3\pi}{4}<x\le n\pi+\pi\end{cases}$$

$\Rightarrow$ $f(x)$ is not differentiable at $x=0,\frac{\pi}{4},\frac{\pi}{2},\frac{3\pi}{4},\pi,\ldots$

$\Rightarrow$ $f(x)$ is not differentiable at $x=\frac{n\pi}{4}$.

8. $\angle AOB = \angle BOC = \angle AOC = 120°$



Area of ΔABC = Area of AOB + Area of BOC + Area of AOC.

$$\Rightarrow \Delta=\frac{1}{2}\left(a'b'\frac{\sqrt3}{2}+b'c'\frac{\sqrt3}{2}+c'a'\frac{\sqrt3}{2}\right)=\frac{\sqrt3}{4}\sum a'b'$$

$$\Rightarrow \sum a'b'=\frac{4}{\sqrt3}\Delta \qquad \text{....(1)}$$

$$\text{In }\Delta AOB,\ \cos\angle AOB=\frac{a'^2+b'^2-c^2}{2a'b'}=-\frac{1}{2}$$

$$\Rightarrow a'^2+b'^2-c^2=-a'b'$$

$$\text{In }\Delta AOC,\ \cos\angle AOC=\frac{a'^2+c'^2-b^2}{2a'c'}=-\frac{1}{2}$$

$$\Rightarrow a'^2+c'^2-b^2=-a'c'$$

$$\text{In }\Delta BOC,\ \cos\angle BOC=\frac{b'^2+c'^2-a^2}{2b'c'}=-\frac{1}{2}$$

$$\Rightarrow b'^2+c'^2-a^2=-b'c'$$

Adding the above three, we get,

$$2\sum a'^2-\sum a'^2=\sum a'^2b'=-\frac{4}{\sqrt3}\Delta,\ \text{using(1)}$$

$$\Rightarrow \sum a'^2=\frac{1}{2}-\left(\sum a'^2-\frac{4}{\sqrt3}\Delta\right) \qquad \text{....(2)}$$

Now $a'+b'+c'=\sqrt{a'^2+b'^2+c'^2+2\sum a'b'}$

$$=\sqrt{\frac{1}{2}\left(\sum a'-\frac{4}{\sqrt3}\Delta\right)+2\times\frac{4}{\sqrt3}\Delta}$$

$$=\sqrt{\frac{\sum a^2}{2}+2\sqrt3\Delta} \qquad \text{...(3)}$$

Again $\frac{a^2+b^2+c^2}{3}\ge(abc)^{2/3}$ (AM ≥ GM)

$$\Rightarrow \frac{\sum a^2}{2}\ge\frac{3}{2}(4\Delta r)^{2/3}\ge\frac{3}{2}(8\Delta R)^{2/3} \qquad (\because R\ge 2r)$$

$$=6(\Delta r)^{2/3}$$

So from (3), we get

$$a'+b'+c'\ge\sqrt{6(\Delta r)^{2/3}+2\sqrt3\Delta}$$

9. $f(x)$is increasing in $(-\infty, 3)$ and decreasing in $(3, \infty)$

Let $x + 1 \le 3$ or $x \le 2$, in this case greatest value of $f(t)$ in $t \in [x, x + 1]$ occurs at right end, namely at $t = x + 1$

Now for $3 < x + 1 \le 4 \Rightarrow x \in (2, 3]$

Greatest value of $f(t)$ is 10

For $4 < x + 1 < 5$, $x \in (3, 4)$

$f(t)$ decreases in $[x, x + 1]$ and greatest values of $f(t)$ is $f(x)$.

$$\text{Here } g(x)=\begin{cases} f(x+1), & 0\le x<2\\ 10, & 2<x\le 3\\ f(x), & 3<x<4\\ 7, & 4\le x\le 6\end{cases}$$

As $\min(x + 3) = 7\ \forall\ x \in [4, 6]$

$$g(x)=\begin{cases}10-(x-2)^2, & 0\le x<2\\ 10, & 2<x\le 3\\ 10-(x-3)^2, & 3<x<4\\ 7, & 4\le x\le 6\end{cases}$$

We will check the behaviour of x at the break point. Function is continuous at all points except at $x = 4$. Let us now check the differentiability at $x = 2, 3$.

$$g'(2+0)=\lim_{h\to0}\frac{g(2+h)-g(2)}{h}=\lim_{h\to0}\frac{10-10}{h}=0$$

$$g'(2-0)=\lim_{h\to0}\frac{g(2-h)-g(2)}{-h}=\lim_{h\to0}\frac{10-h^2-10}{-h}=0$$

Therefore $g(x)$ is differentiable at $x = 2$

At $x = 3$,

$$g'(3-0)=\lim_{h\to 0}\frac{g(3-h)-g(3)}{-h}=\lim_{h\to 0}\frac{10-10}{h}=0$$

$$g'(3+h)=\lim_{h\to 0}\frac{g(3+h)-g(3)}{h}=\lim_{h\to 0}\frac{10-h^2-10}{-h}=0$$

It is also differentiable at $x = 3$

Since $g(x)$ is discontinuous at $x = 4$.

∴ It is non-differentiable also.

10. $\sin(A - B) = \sin A \cos B - \cos A \sin B$

Substituting values of $\cos A$ and $\sin A$ from the given equations, we get

$$\sin(A-B)=\frac{\sin B\cos^2 B\cos B-(\cos B+\cos^2 B)\sin B}{\sqrt{2}}$$

$$=\frac{-\sin B\cos B}{\sqrt{2}}$$

Squaring and adding given equations

$(\cos B + \cos^3 B)^2 + (\sin B - \sin^3 B)^2 = 2$

$$\Rightarrow \cos^2 B+\cos^6 B+2\cos^4 B+\sin^2 B+\sin^6 B-2\sin^4 B=2$$

$$\Rightarrow 1+2(\cos^2 B+\sin^2 B)+(\sin^4 B+\cos^4 B-\sin^2 B\cos^2 B)=2$$

$$\Rightarrow 1+2\cos^2 B-2\sin^2 B+1-3\sin^2 B\cos^2 B=2$$

$\Rightarrow 3\cos^2 2B + 8\cos 2B - 3 = 0$

$$\Rightarrow \cos 2B=\frac{1}{3} \Rightarrow \sin 2B=\pm\frac{2\sqrt{2}}{3}$$

$$\Rightarrow \sin(A-B)=-\frac{\sin 2B}{2\sqrt{2}}=\pm\frac{1}{3}$$

11. Equation of the tangent to the ellipse

$$a^2x^2+\frac{y^2}{2}=1 \text{ is } ax\cos\theta+\frac{1}{\sqrt{2}}y\sin\theta=1$$

Now homogenising the hyperbola by the given tangent we get

$$\Rightarrow \frac{x^2}{a^2}-2y^2=\left(ax\cos\theta+\frac{y\sin\theta}{\sqrt{2}}\right)^2$$

$$\Rightarrow \left(\frac{1}{a^2}-a^2\cos^2\theta\right)x^2+\left(-2-\frac{\sin^2\theta}{2}\right)y^2$$

$$-a\sqrt{2}\sin\theta\cos\theta\ xy=0$$

Since this subtends a right angle at the centre so,

$$\frac{1}{a^2}-a^2\cos^2\theta-2-\frac{\sin^2\theta}{2}=0$$

$$\Rightarrow \frac{1}{a^2}-a^2+\left(a^2-\frac{1}{2}\right)\sin^2\theta-2=0$$

$$\Rightarrow \sin^2\theta=\frac{2+a^2-\frac{1}{a^2}}{a^2-\frac{1}{2}} \Rightarrow 0\le\frac{2+a^2-\frac{1}{a^2}}{a^2-\frac{1}{2}}\le 1$$

$$\Rightarrow \frac{a^2-\frac{2}{5}}{a^2-\frac{1}{2}}\le 0\Rightarrow a\in\left[-\sqrt{\frac{1}{2}},-\sqrt{\frac{2}{5}}\right]\cup\left(\sqrt{\frac{2}{5}},\sqrt{\frac{1}{2}}\right) \quad ...(1)$$

$$\text{Again } \frac{2+a^2-\frac{1}{a^2}}{a^2-\frac{1}{2}}\ge 0 \Rightarrow (a^4+2a^2-1)\left(a^2-\frac{1}{2}\right)\ge 0$$

$$\Rightarrow \left(a-\frac{1}{\sqrt{2}}\right)\left(a+\frac{1}{\sqrt{2}}\right)(a-\sqrt{\sqrt{2}-1})(a-\sqrt{\sqrt{2}-1})\ge 0$$

$$\Rightarrow a\in\left(-\infty,-\frac{1}{\sqrt{2}}\right)\cup\left[-\sqrt{\sqrt{2}-1},\sqrt{\sqrt{2}-1}\right]\cup\left(\frac{1}{\sqrt{2}},\infty\right) \quad ...(2)$$

Now taking intersection (1) and (2), we get

$$a\in\left[-\sqrt{\sqrt{2}-1},-\sqrt{\frac{2}{5}}\right]\cup\left[\sqrt{\frac{2}{5}},\sqrt{\sqrt{2}-1}\right].$$

12. (a) Let $a = \tan\theta$

$$\Rightarrow \left(\frac{i\tan\theta-1}{i\tan\theta+1}\right)^n=\left(\frac{i\sin\theta-\cos\theta}{i\sin\theta+\cos\theta}\right)^n$$

$$\Rightarrow \left(\frac{-i^3\sin\theta+i^2\cos\theta}{-i^3\sin\theta-i^2\cos\theta}\right)^n=\left(\frac{-i^2(\cos\theta-i\sin\theta)}{-i^2(\cos\theta+i\sin\theta)}\right)^n$$

$$=(-1)^n\frac{e^{-in\theta}}{e^{in\theta}}=(-1)^n(e^{-2in\theta})$$

$$\text{Therefore, } (-1)^n e^{-2ni\tan^{-1}a}.e^{2ni\tan^{-1}a}=(-1)^n$$

(b) $q+ir=(a+p)z \Rightarrow z=\dfrac{q+ir}{a+p}$

$$\Rightarrow \frac{iz}{1}=\frac{-r+iq}{a+p}$$

By componendo and dividendo, we get

$$\frac{1+iz}{1-iz}=\frac{a+p-r+iq}{a+p+r-iq}=\frac{(p+iq)+(a-r)}{(p-iq)+(a+r)}$$

Now $p^2 + q^2 = a^2 - r^2$

$(p + iq)(p - iq) = (a + r)(a - r)$

$$\Rightarrow \frac{p+iq}{a+r}=\frac{a-r}{p-iq}=\frac{p+iq+a-r}{a+r+p-iq}$$

From $\frac{p-iq}{a+r}=\frac{1-iz}{1-iz}$

similarly, $\frac{r+ip}{a+q}=\frac{i(1-z)}{(1+z)}$.

13. Given $ABCDE$ is a regular pentagon

Since ABC and O form a parallelogram and sides of regular pentagon are equal $|\vec{a}|=|\vec{b}|=r$.

$\overrightarrow{OB}=\vec{a}+\vec{b}$ ...(1)

Since $\angle ABC=\frac{3\pi}{5}$, each interior angle of a regular pentagon.

$\angle AOC=\angle ABC=\frac{3\pi}{5}$ (from (1))

Now, $OA = BC$ and $OC = AB$

So, $OABC$ is a rhombus.

Hence $\angle OAB=\angle OCB=\frac{2\pi}{5}$

Let $OD=\lambda\vec{a}$, $OE=\lambda\vec{b}$

$\angle EOD=\frac{3\pi}{5}$ as DO and OA, and EO and OC are collinear

In ΔODC

As $OC = BC$ and $DC = CB$ (sides of a regular pentagon)

$\angle DOC=\pi-\frac{3\pi}{5}=\frac{2\pi}{5}$

Since $|\vec{a}|=|r|=|\vec{b}|$

Therefore, $|\lambda\vec{a}|=2|r|\cos\frac{2\pi}{5} \Rightarrow \lambda=-2\cos\frac{2\pi}{5}$

Now, area of pentagon

= Area of $OABC$ + area of ODC + area of OEA + area of OED

$$=|\vec{a}\times\vec{b}|+\frac{|\vec{b}\times\lambda\vec{a}|}{2}+\frac{|\lambda||\vec{a}\times\lambda\vec{b}|}{2}+\frac{|\lambda||\vec{b}\times\vec{a}|}{2}$$

$$=|\vec{a}\times\vec{b}|\left[1+\frac{|\lambda|}{2}+\frac{|\lambda|}{2}+\frac{\lambda^2}{2}\right]=|\vec{a}\times\vec{b}|\left[1+|\lambda|+\frac{\lambda^2}{2}\right]$$

14. 1, 2, 2, 3, 3, 3, 4, 4, 4, 4,....

Let us write the terms in the groups as under, 1, (2, 2), (3, 3, 3), (4,4,4,4),... consisting of 1,2,3,4,... terms. Let 2000th term falls in n^{th} group.

$\Rightarrow \quad \frac{(n-1)n}{2}<2000 \le \frac{n(n+1)}{2}$

$\Rightarrow \; n(n-1) < 4000 \le n(n+1)$

Let us consider, $n(n-1) < 4000$

$\Rightarrow \; n^2 - n - 4000 < 0$

$\Rightarrow \; n<\frac{1+\sqrt{16001}}{2} \quad \Rightarrow \; n<64$

From $n(n+1) \ge 4000$, $n^2 + n - 4000 \ge 0 \Rightarrow n \ge 63$

$\Rightarrow \; n = 63$

That means 2000th term falls in 63th group. That also means that 2000 terms is 63.

Now total number of terms upto 62th group

$=\frac{62.63}{2}=1953$

$\Rightarrow \; 1^2 + 2^2 + ... + 62^2 + 63\ (2000 - 1953)$

$=\frac{62(63)125}{6}+63.47=84336$

15. Let $I=\sum_{0\le i<}\sum_{j\le n}\left(\frac{i}{{}^nC_i}+\frac{j}{{}^nC_j}\right)$

$$=\sum_{0\le i<}\sum_{j\le n}\left(\frac{n-i}{{}^nC_{n-i}}+\frac{n-j}{{}^nC_{n-j}}\right)$$

$$=n\sum_{0\le i<}\sum_{j\le n}\left(\frac{1}{{}^nC_i}+\frac{1}{{}^nC_j}\right)-I$$

$$I=\frac{n}{2}\sum_{0\le i<}\sum_{j\le n}\left(\frac{1}{{}^nC_i}+\frac{1}{{}^nC_j}\right)=\frac{n}{2}\left(\sum_{r=0}^{n-1}\frac{n-r}{{}^nC_r}+\sum_{r=1}^{n}\frac{r}{{}^nC_r}\right)$$

$$=\frac{n}{2}\left(\sum_{r=0}^{n}\frac{n}{{}^nC_r}\right)=\frac{n^2}{2}.a$$

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PRACTICE PAPER 2018
JEE MAIN

Exam Dates
OFFLINE : 8th April
ONLINE : 15th & 16th April

1. Total number of real values of x, such that $\frac{(12+x)^{1/7}}{x}+\frac{(12+x)^{1/7}}{12}=\frac{64}{3}x^{1/7}$ is equal to
(a) one (b) two
(c) zero (d) none of these

2. If the equations $x^2 + ax + 12 = 0$, $x^2 + bx + 15 = 0$ and $x^2 + (a + b)x + 36 = 0$ have a common positive root then ordered pair (a, b) is
(a) $(-6, -8)$ (b) $(-7, -8)$
(c) $(-6, -7)$ (d) $(-7, -9)$

3. If $x_1, x_2, ..., x_n$ are in H.P., then $\sum_{r=1}^{n-1} x_r x_{r+1}$ is equal to
(a) $(n - 1)x_1x_n$ (b) nx_1x_n
(c) $(n + 1)x_1x_n$ (d) none of these

4. If $|z| \le 4$, then the maximum value of $|iz + 3 - 4i|$ is equal to
(a) 2 (b) 4 (c) 3 (d) 9

5. The value of $\sum_{r=0}^{n}(r)\,{}^{2n}C_r$ is equal to
(a) $2n(2^{2n-2} + {}^{2n-1}C_{n-1})$
(b) $n(2^{2n-2} + {}^{2n-1}C_n)$
(c) $2n(2^{2n-2} + {}^{2n-1}C_{n+1})$
(d) none of these

6. Total number of ways, in which 22 different books can be given to 5 students, so that two students get 5 books each and all the remaining students get 4 books each, is equal to
(a) $\frac{22!}{3!2!5!(4!)^3}$ (b) $\frac{22!}{(3!)^2\cdot 2!\cdot 5!}$
(c) $\frac{22!}{3!\,2!\,5!4!}$ (d) None of these

7. A team of four students is to be selected from a total of 12 students. Total number of ways in which team can be selected such that two particular students refuse to be together and other two particular students wish to be together only, is equal to
(a) 220 (b) 182
(c) 226 (d) none of these

8. In triangle ABC, the value of $\begin{vmatrix} e^{-i2A} & e^{iC} & e^{iB} \\ e^{iC} & e^{-i2B} & e^{iA} \\ e^{iB} & e^{iA} & e^{-i2C} \end{vmatrix}$ where $i=\sqrt{-1}$, is equal to
(a) -2 (b) -4
(c) -3 (d) none of these

9. Let $\vec{a},\vec{b},\vec{c}$ be three unit vectors such that $\vec{a}+5\vec{b}+3\vec{c}=\vec{0}$ then $\vec{a}\cdot(\vec{b}\times\vec{c})$ is equal to
(a) $\vec{a}\cdot\vec{b}$ (b) $\vec{a}\cdot(\vec{b}+2\vec{c})$
(c) $\vec{b}\cdot(\vec{a}+\vec{c})$ (d) none of these

10. Consider the set of integers {10, 11, 12, ..., 98, 99}. By seeing the number a person A will laugh if the product of the digits of the number is 12. He chooses three numbers one at a time from this set of integers, randomly and with replacement. The probability that he will laugh atleast once, is equal to
(a) $1-\left(\frac{43}{45}\right)^3$ (b) $1-\left(\frac{3}{5}\right)^3$
(c) $1-\left(\frac{4}{25}\right)^3$ (d) none of these

11. The probability that an archer hits the target when it is windy is equal to 2/5. When it is not windy his probability of hitting the target is 7/10. On any shot the probability of gust of wind is 3/10. The probability that there is no gust of wind on the occasion when he missed the target, is equal to
(a) 5/13 (b) 18/39 (c) 7/13 (d) 23/39

12. If A and B are square matrices such that $AB = B$ and $BA = A$, then $A^2 + B^2$ is always equal to
(a) $2AB$ (b) $2BA$ (c) $A + B$ (d) AB

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13. Range of $f(x)=\cos^{-1}\left(\frac{1+x^2}{2x}\right)+\sqrt{2-x^2}$ is

(a) $\left\{0, 1+\frac{\pi}{2}\right\}$ (b) $\{0, 1+\pi\}$

(c) $\left\{1, 1+\frac{\pi}{2}\right\}$ (d) $\{1, 1+\pi\}$

14. If $[\cos^{-1}x]+[\cot^{-1}x]=0$, where [.] denotes the greatest integer function, then complete set of values of x is

(a) $(\cos 1, 1]$ (b) $(\cos 1, \cot 1)$

(c) $(\cot 1, 1]$ (d) $[0, \cot 1)$

15. $\lim_{x\to 1}\frac{(x-1)(x^2-1)(x^3-1)(x^4-1)}{((x-1)(x^2-1))^2}$ is equal to

(a) 2 (b) 3

(c) 6 (d) none of these

16. $f(x)=[\sin x]+[\cos x]$, $x\in[0, 2\pi]$, where [.] denotes the greatest integer function. Total number of points where $f(x)$ is non-differentiable is equal to

(a) 2 (b) 3 (c) 5 (d) 4

17. $f(x)=x^2-4|x|$ and

$g(x)=\begin{cases}\min\{f(t):-6\le t\le x\}, x\in[-6,0]\\ \max\{f(t):0<t\le x\}, x\in(0,6]\end{cases}$ then $g(x)$ has

(a) exactly one point of local minima

(b) exactly one point of local maxima

(c) no point of local maxima but exactly one point of local minima

(d) neither a point of local maxima nor minima

18. $\int \sin x\cdot\cos x\cdot\cos 2x\cdot\cos 4x\cdot\cos 8x\,dx$ is equal to

(a) $\frac{1}{96}\cos 16x+c$ (b) $-\frac{1}{256}\cos 16x+c$

(c) $-\frac{1}{16}\cos 16x+c$ (d) none of these

19. $\int_0^{\frac{3\pi}{2}}\sin\left[\frac{2x}{\pi}\right]dx$ where [.] denotes the greatest integer function, is equal to

(a) $\frac{\pi}{2}(\sin 1+\cos 1)$ (b) $\frac{\pi}{2}(\sin 1-\sin 2)$

(c) $\frac{\pi}{2}(\sin 1-\cos 1)$ (d) $\frac{\pi}{2}(\sin 1+\sin 2)$

20. Value of positive real parameter 'a' such that the area of the region bounded by the parabola $y=x-ax^2$, $ay=x^2$ attains its maximum value, is equal to

(a) 1/2 (b) 2

(c) 1 (d) none of these

21. General solution of $x\frac{dy}{dx}+y=y^2x^3\cos x$, is

(a) $\frac{1}{xy}+x\sin x-\cos x=C$

(b) $\frac{1}{xy}-x\sin x+\cos x=C$

(c) $\frac{1}{xy}+x\cos x-\sin x=C$

(d) $\frac{1}{xy}+x\sin x+\cos x=C$

22. A variable line is drawn through the intersection point of the lines $3x+4y-12=0$ and $x+2y-5=0$, meeting the co-ordinate axes at the points A and B. Locus of midpoint of segment AB is

(a) $4x+3y=4xy$ (b) $3x+4y=3xy$

(c) $3x+4y=4xy$ (d) none of these

23. If $x-2y+4=0$ and $2x+y-5=0$ are the sides of an isosceles triangle having area 10 sq. units, then equation of third side is

(a) $x+3y=19$ (b) $3x-y+11=0$

(c) $x-3y=10$ (d) $3x-y+15=0$

24. The circle $x^2+y^2-4x-4y+4=0$ is inscribed in a variable triangle OAB. Sides OA and OB lie along the x and y-axis respectively, where 'O' is the origin. Locus of midpoint of side AB is $x+y-xy+\lambda\sqrt{x^2+y^2}=0$, then λ is equal to

(a) –1 (b) 1 (c) –2 (d) 2

25. PA and PB are tangents drawn to $y^2=4ax$ from arbitrary point P. If the angle between tangents is $\pi/4$, then locus of point P is

(a) $y^2=x^2+a^2+6ax$ (b) $y^2=x^2-a^2+6ax$

(c) $y^2=x^2+a^2-6ax$ (d) $y^2=x^2-a^2-6ax$

26. The equation of common tangents of the curves $x^2+4y^2=8$ and $y^2=4x$ are

(a) $y-2x-4=0$, $y+2x+4=0$

(b) $y-2x-2=0$, $y+2x+2=0$

(c) $2y-x-4=0$, $2y+x+4=0$

(d) $2y-x-2=0$, $2y+x+2=0$

27. Length of the latus rectum of the hyperbola $xy = c^2$, is equal to

(a) $2c$ (b) $\sqrt{2}c$ (c) $2\sqrt{2}c$ (d) $4c$

28. The equation of the plane passing through the points $(2, -1, 0)$, $(3, -4, 5)$ and parallel to the line $2x = 3y = 4z$ is

(a) $125x - 90y - 79z = 340$
(b) $32x - 21y - 36z = 85$
(c) $73x + 61y - 22z = 85$
(d) $29x - 27y - 22z = 85$

29. Reflection of the line $\frac{x-1}{-1} = \frac{y-2}{3} = \frac{z-4}{1}$ in the plane $x + y + z = 7$ is

(a) $\frac{x-1}{3} = \frac{y-2}{1} = \frac{z-4}{1}$
(b) $\frac{x-1}{-3} = \frac{y-2}{-1} = \frac{z-4}{1}$
(c) $\frac{x-1}{-3} = \frac{y-2}{1} = \frac{z-4}{-1}$
(d) $\frac{x-1}{3} = \frac{y-2}{1} = \frac{z-4}{-1}$

30. If x, y, z are in A.P. then $\frac{\sin x - \sin z}{\cos z - \cos x}$ is equal to

(a) $\tan y$ (b) $\cot y$ (c) $\sin y$ (d) $\cos y$

31. The minimum value of $27^{\cos x} + 81^{\sin x}$ is equal to

(a) $\frac{2}{3\sqrt{3}}$ (b) $\frac{1}{3\sqrt{3}}$
(c) $\frac{2}{9\sqrt{3}}$ (d) none of these

32. $\sum_{r=1}^{n} \sin^{-1}\left(\frac{\sqrt{r}-\sqrt{r-1}}{\sqrt{r(r+1)}}\right)$ is equal to

(a) $\tan^{-1}(\sqrt{n}) - \frac{\pi}{4}$ (b) $\tan^{-1}(\sqrt{n+1}) - \frac{\pi}{4}$
(c) $\tan^{-1}(\sqrt{n})$ (d) none of these

33. Equation of the plane such that foot of altitude drawn from $(-1, 1, 1)$ to the plane has the coordinate $(3, -2, -1)$, is

(a) $x + y + z = 0$ (b) $4x - 3y - 2z = 20$
(c) $3x + y - z = 8$ (d) $4x + 3y - z = 7$

34. If in triangle ABC, $\angle A = \pi/2$ then the value of $r + 2R$ is equal to

(a) $\frac{1}{2}(a + b - c)$ (b) $\frac{1}{2}(b + c - a)$
(c) $\frac{1}{2}(a + c - b)$ (d) $\frac{1}{2}(a + b + c)$

35. If a, b, c are the sides of a triangle ABC then $\sqrt{a} + \sqrt{b} - \sqrt{c}$ is always

(a) negative (b) positive
(c) non-negative (d) non-positive

36. $\int \tan^{-1}\sqrt{x}\,dx$ is equal to

(a) $(x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$
(b) $x\tan^{-1}\sqrt{x} - \sqrt{x} + C$
(c) $\sqrt{x} - x\tan^{-1}\sqrt{x} + C$
(d) $\sqrt{x} - (x+1)\tan^{-1}\sqrt{x} + C$

37. If the curves $\frac{x^2}{a^2} + \frac{y^2}{4} = 1$ and $y^2 = 16x$ intersect at right angle, then

(a) $a = \pm 1$ (b) $a = \pm\sqrt{3}$
(c) $a = \pm\frac{1}{\sqrt{3}}$ (d) $a = \pm\sqrt{2}$

38. If $f(x) = 2e^x - ae^{-x} + (2a + 1)x - 3$ is increasing for all values of x, then

(a) $a \in (-\infty, \infty)$ (b) $a \in (-\infty, 0]$
(c) $a \in [0, \infty)$ (d) None of these

39. Domain of $f(x) = \sqrt{[x] - 1 + x^2}$; where [.] denotes the greatest integer function, is

(a) $(-\infty, -2) \cup [1, \infty)$ (b) $(-\infty, -\sqrt{2}) \cup [1, \infty)$
(c) $(-\infty, -3) \cup [1, \infty)$ (d) $(-\infty, -\sqrt{3}] \cup [1, \infty)$

40. $\sum_{r=1}^{n} \frac{r}{(r+1)!}$ is equal to

(a) $1 - \frac{1}{(n+1)!}$ (b) $1 - \frac{1}{n!}$
(c) $1 - \frac{1}{(n-1)!}$ (d) none of these

SOLUTIONS

1. **(a) :** $\frac{(12+x)^{1/7}}{x} + \frac{(12+x)^{1/7}}{12} = \frac{64}{3}x^{1/7}$

$\Rightarrow (12 + x)(12 + x)^{1/7} = 64 \cdot 4 \cdot x \cdot x^{1/7} = 2^8 \cdot x^{8/7}$

$\Rightarrow 12 + x = x(2^8)^{7/8} \Rightarrow x = \frac{12}{127}$

2. **(b) :** We have, $x^2 + ax + 12 = 0$ ...(i)

$x^2 + bx + 15 = 0$ ...(ii)

$x^2 + (a + b)x + 36 = 0$ ...(iii)

Adding (i) & (ii) we get

$2x^2 + (a + b)x + 27 = 0$ ...(iv)

Now, subtracting (iii) from (iv), we get

$x^2 - 9 = 0 \quad \Rightarrow \quad x = 3, -3.$

Thus, common positive root is 3.

$\therefore \quad 9 + 12 + 3a = 0 \quad \Rightarrow \quad a = -7$

and $9 + 3b + 15 = 0 \quad \Rightarrow \quad b = -8$

Thus ordered pair (a, b) is $(-7, -8)$.

3. (a) : Clearly, $\frac{1}{x_1}, \frac{1}{x_2}, \ldots, \frac{1}{x_n}$ will be in A.P.

$$\Rightarrow \frac{1}{x_2} - \frac{1}{x_1} = \frac{1}{x_3} - \frac{1}{x_2} = \ldots = \frac{1}{x_{r+1}} - \frac{1}{x_r} = \ldots$$

$$= \frac{1}{x_n} - \frac{1}{x_{n-1}} = \lambda \text{ (say)}$$

$$\Rightarrow \sum_{r=1}^{n-1} x_r x_{r+1} = \frac{1}{\lambda} \sum_{r=1}^{n-1} (x_r - x_{r+1}) = \frac{1}{\lambda}(x_1 - x_n)$$

Now, $\frac{1}{x_n} = \frac{1}{x_1} + (n-1)\lambda \Rightarrow \frac{x_1 - x_n}{x_1 x_n} = (n-1)\lambda$

$$\Rightarrow \sum_{r=1}^{n-1} x_r x_{r+1} = (n-1)x_1 x_n$$

4. (d) : $|iz + 3 - 4i| \le |iz| + |3 - 4i| = |z| + 5 \le 9$

5. (d) : We have, $\sum_{r=0}^{n} r \, . \, {}^{2n}C_r = \sum_{r=0}^{n} 2n \, . \, {}^{2n-1}C_{r-1}$

$= 2n({}^{2n-1}C_0 + {}^{2n-1}C_1 + \ldots + {}^{2n-1}C_{n-1})$

$= 2n(\frac{1}{2}(2^{2n-1})) = 2n(2^{2n-2})$

6. (a) : It is same as dividing 22 books in 5 packets such that two packets have 5 books each and remaining three have four books each and then distributing these packets.

Thus, total ways of distributing these books

$$= \frac{22!}{(5!)^2 \cdot (4!)^3 \cdot 2!3!} . (5!) = \frac{22!}{2!3!5!(4!)^3}$$

7. (c) : Let S_1 and S_2 refuse to be together and S_3 and S_4 want to be together only.

Total ways when S_3 and S_4 are selected

$= ({}^8C_2 + {}^2C_1 \cdot {}^8C_1) = 44$

Total ways when S_3 and S_4 are not selected

$= ({}^8C_4 + {}^2C_1 \cdot {}^8C_3) = 182$

Thus total ways = 44 + 182 = 226.

8. (b) : Given determinant

$$= e^{-i2A}(e^{-i2(B+C)} - e^{i2A}) - e^{iC}(e^{-iC} - e^{i(A+B)}) + e^{iB}(e^{i(A+C)} - e^{-iB})$$

$= e^{-i2(A+B+C)} - 1 - 1 + e^{i(A+B+C)} + e^{i(A+B+C)} - 1$

$= e^{-i2\pi} - 1 - 1 + e^{i\pi} + e^{i\pi} - 1$

$= 1 - 1 - 1 - 1 - 1 - 1 = -4$

9. (d) : $\vec{a} + 5\vec{b} + 3\vec{c} = \vec{0}$

Thus $\vec{a}, \vec{b}, \vec{c}$ are coplanar.

Hence $\vec{a} \cdot (\vec{b} \times \vec{c}) = 0$

10. (a) : Person will laugh if the selected number is 43 or 34 or 62 or 26.

Thus, probability that person will not laugh on seeing the number $= \frac{86}{90} = \frac{43}{45}$

Thus, required probability $= 1 - \left(\frac{43}{45}\right)^3$

11. (c) : Let E_1 : There is a gust of wind.

E_2 : There is no gust of wind.

A : archer misses the target.

Now, $P(A) = P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)$

$$= \frac{3}{10} \cdot \frac{3}{5} + \frac{7}{10} \cdot \frac{3}{10} = \frac{39}{100}$$

$\therefore$ Required probability $= P(E_2/A) = \frac{P(E_2 \cap A)}{P(A)}$

$$= \frac{P(E_2) \cdot P(A/E_2)}{P(A)} = \frac{\frac{7}{10} \cdot \frac{3}{10}}{\frac{39}{100}} = \frac{7}{13}$$

12. (c) : Given, $AB = B$, $BA = A$

Now, $A^2 + B^2 = AA + BB$

$= A(BA) + B(AB) = (AB)A + (BA)B$

$= BA + AB = A + B$

13. (d) : $\cos^{-1}\left(\frac{1+x^2}{2x}\right)$ is defined if $\left|\frac{1+x^2}{2x}\right| \le 1$

$\Rightarrow \quad x \ne 0$

$\Rightarrow \quad 1 + x^2 - 2|x| \le 0 \quad \Rightarrow \quad (|x| - 1)^2 \le 0$

$\Rightarrow \quad x = 1, -1$

Thus, domain of $f(x)$ is $\{1, -1\}$.

Hence, range is $\{1, 1 + \pi\}$

14. (c) : We have, $[\cos^{-1}x] \ge 0 \; \forall \; x \in [-1, 1]$

and $[\cot^{-1}x] > 0 \; \forall \; x \in R$

Now, $[\cos^{-1}x] + [\cot^{-1}x] = 0$

$\Rightarrow \quad [\cos^{-1}x] = [\cot^{-1}x] = 0$

If $[\cos^{-1}x] = 0$, then $x \in (\cos 1, 1]$

If $[\cot^{-1}x] = 0$, then $x \in (\cot 1, \infty)$

$\Rightarrow \quad x \in (\cot 1, 1]$

15. (c) : We have, $\dfrac{(x-1)(x^2-1)(x^3-1)(x^4-1)}{((x-1)(x^2-1))^2}$

$$= \frac{(x^3-1)(x^4-1)}{(x-1)(x^2-1)} = (x^2+x+1)(x^2+1)$$

$$\therefore \lim_{x\to 1} \frac{(x-1)(x^2-1)(x^3-1)(x^4-1)}{((x-1)(x^2-1))^2}$$

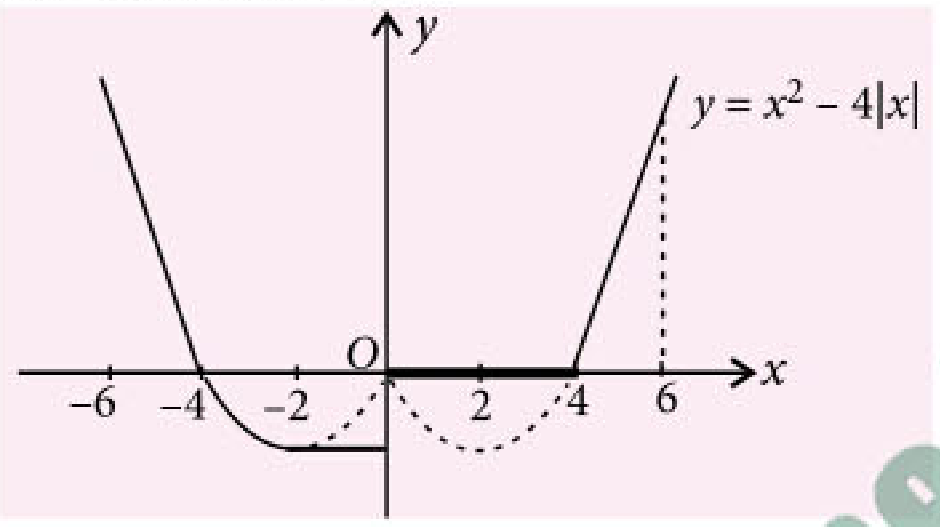
$$= \lim_{x\to 1}(x^2+x+1)(x^2+1) = 6$$

16. (c) : $[\sin x]$ is discontinuous at $x = \pi/2, \pi, 2\pi$ and $[\cos x]$ is discontinuous at $x = 0, \pi/2, 3\pi/2, 2\pi$.

Thus, $f(x)$ is discontinuous at $x = 0, \dfrac{\pi}{2}, \pi, 3\pi/2, 2\pi$.

Thus, $f(x)$ is discontinuous and hence non-differentiable at 5 points.

17. (d) : Bold line represents the graph of $y = g(x)$. Clearly, $g(x)$ has neither a point of local maxima nor a point of local minima.



18. (b) : $\int \sin x \cdot \cos x \cdot \cos 2x \cdot \cos 4x \cdot \cos 8x \; dx$

$$= \frac{1}{2}\int \sin 2x \cdot \cos 2x \cdot \cos 4x \cdot \cos 8x \, dx$$

$$= \frac{1}{4}\int \sin 4x \cdot \cos 4x \cdot \cos 8x \, dx$$

$$= \frac{1}{8}\int \sin 8x \cdot \cos 8x \; dx = \frac{1}{16}\int \sin 16x \; dx$$

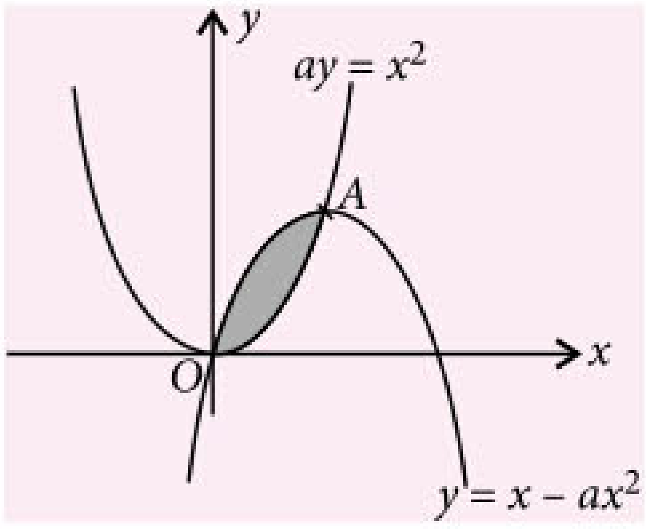
$$= \frac{-1}{256}\cos 16x + C$$

19. (d) : $\displaystyle\int_0^{3\pi/2} \sin\left[\frac{2x}{\pi}\right]dx$

$$= \int_0^{\pi/2} \sin\left[\frac{2x}{\pi}\right]dx + \int_{\pi/2}^{\pi} \sin\left[\frac{2x}{\pi}\right]dx + \int_{\pi}^{3\pi/2} \sin\left[\frac{2x}{\pi}\right]dx$$

$$= 0 + \sin 1 \int_{\pi/2}^{\pi} dx + \sin 2 \int_{\pi}^{3\pi/2} dx \; = \frac{\pi}{2}(\sin 1 + \sin 2)$$

20. (c) : At point A,



$$\frac{x^2}{a} = x - ax^2$$

$$\Rightarrow \; x = 0, \; x = \frac{a}{1+a^2}$$

$\therefore$ Required area,

$$\Delta = \int_0^{a/1+a^2}\left(x - ax^2 - \frac{x^2}{a}\right)dx$$

$$\Rightarrow \; \frac{d\Delta}{da} = \int_0^{a/1+a^2}\left(-x^2 + \frac{x^2}{a^2}\right)dx + 0 \cdot \frac{d}{da}\left(\frac{a}{1+a^2}\right)$$

$$= \frac{(1-a^2)}{3a^2}x^3\Big|_0^{a/1+a^2} = \frac{a(1-a^2)}{3(1+a^2)^3} = \frac{a(1-a)(1+a)}{3\,(1+a^2)^3}$$

$$\Rightarrow \; \frac{d\Delta}{da} > 0 \;\forall\; a \in (0,1)$$

and $\dfrac{d\Delta}{da} < 0 \;\forall\, a \in (1,\infty)$

Thus, $a = 1$ is the point of maxima for Δ.

21. (d) : Given, $x.\dfrac{dy}{dx} + y = y^2 \cdot x^3 \cdot \cos x$

$$\Rightarrow \; \frac{(xdy + ydx)}{x^2y^2} = x \cdot \cos x dx$$

$$\Rightarrow \; \frac{d(xy)}{x^2y^2} = x \cdot \cos x dx \;\Rightarrow\; -\frac{1}{xy} = x\sin x + \cos x + C$$

22. (c) : Equation of any line drawn through the intersection point of the given lines is,

$$3x + 4y - 12 + \lambda(x + 2y - 5) = 0$$

$$\Rightarrow \; A \equiv \left(\frac{5\lambda+12}{3+\lambda}, 0\right), \; B \equiv \left(0, \frac{5\lambda+12}{2\lambda+4}\right)$$

If midpoint of AB is $C(h, k)$, then

$$2h = \frac{5\lambda+12}{3+\lambda}, \; 2k = \frac{5\lambda+12}{2\lambda+4}$$

$$\Rightarrow \; \frac{h}{k} = \frac{2(\lambda+2)}{3+\lambda} \;\Rightarrow\; \lambda = \frac{4k-3h}{(h-2k)}$$

Thus, $2h = \dfrac{12 + \dfrac{5(4k-3h)}{(h-2k)}}{3 + \dfrac{4k-3h}{h-2k}}$

Thus locus is, $3x + 4y = 4xy$

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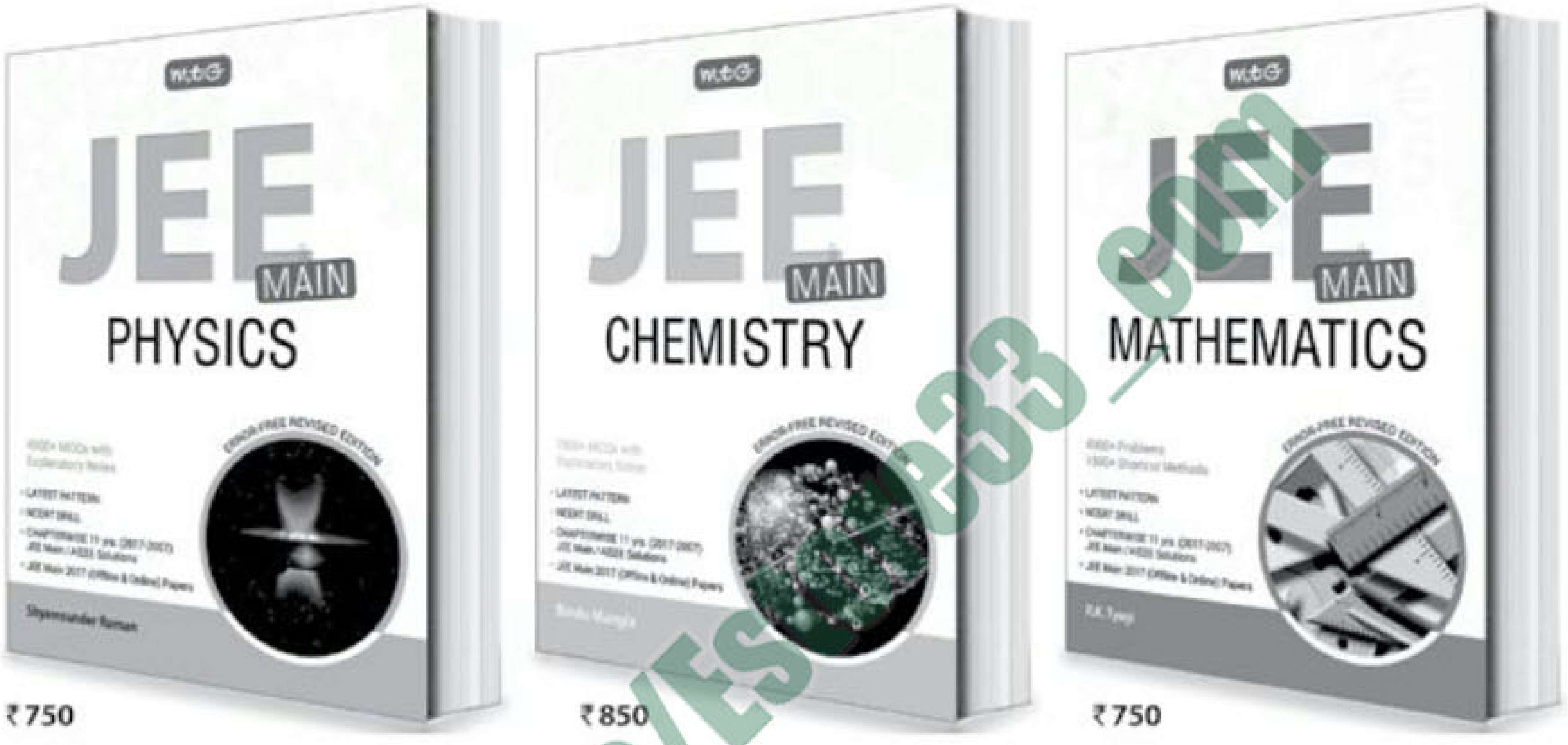

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CONCEPT MAP

STRAIGHT LINES

Class XI

General Equation

$ax + by + c = 0, |a| + |b| \neq 0$

Slope $= m = \tan\theta = \frac{a}{b}$

x-intercept $= \frac{-c}{a}$, y-intercept $= \frac{-c}{b}$

Family of Lines

Let $L_1 = a_1x + b_1y + c_1 = 0$ and
$L_2 = a_2x + b_2y + c_2 = 0$ be two lines then,
$L_1 + \lambda L_2 = 0$ or $L_2 + \lambda L_1 = 0$ gives family of lines passing through the point of intersection.

Concurrent lines :

- If $L_r = a_r x + b_r y + c_r = 0$ (where $r = 1, 2, 3$)

  then $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$
- $\lambda_1 L_1 + \lambda_2 L_2 + \lambda_3 L_3 = 0$, where $\lambda_1, \lambda_2, \lambda_3 \in R$, not all zero's

Perpendicular Distance

- Distance of a point (x_1, y_1) from line $ax + by + c = 0$ is given by $\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right|$.
- Distance between two parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is given by $\left|\frac{c_1 - c_2}{\sqrt{a^2+b^2}}\right|$

A Line and Two Points

$L : ax + by + c = 0$, $A(x_1, y_1)$ and $B(x_2, y_2)$.

- A and B are on the same side of L if

  $$\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c} > 0$$
- A and B are on the opposite sides of L if

  $$\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c} < 0$$
- AB is divided by L in the ratio

  $$\frac{-(ax_1 + by_1 + c)}{(ax_2 + by_2 + c)}$$

Standard Forms

- Slope-intercept form : $y = mx + c$
  Here, m is the slope making an intercept c on y-axis.
- Point-slope form : $\frac{y - y_1}{x - x_1} = m$

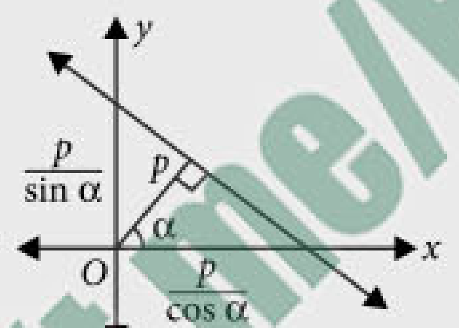
  Here, m is the slope and (x_1, y_1) be given point on the line.
- Two point form : $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

  Here (x_1, y_1) & (x_2, y_2) are two given points on the line.
- Intercept form : $\frac{x}{a} + \frac{y}{b} = 1$

  Here a and b represents x and y intercepts respectively of the line.
- Determinant form : $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$, where (x_1, y_1) and (x_2, y_2) are any two points on the line.
- Perpendicular form (or) Normal form of $ax + by + c = 0$ is
  $x\cos\alpha + y\sin\alpha = p$ where,

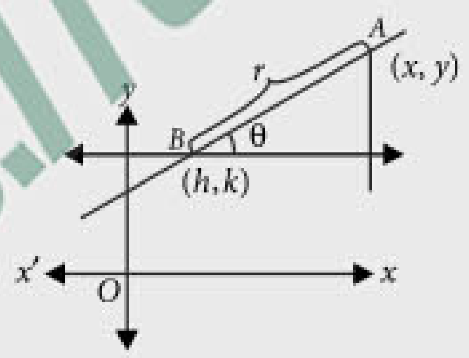
  $\sin\alpha = \frac{b}{\sqrt{a^2+b^2}}$,

  $\cos\alpha = \frac{a}{\sqrt{a^2+b^2}}, p = \frac{-c}{\sqrt{a^2+b^2}} > 0$



- Parametric form : $\frac{x - h}{\cos\theta} = \frac{y - k}{\sin\theta} = r$

  Where $(x, y) = (h \pm r\cos\theta, k - r\sin\theta)$



Angle Between Two Lines

- For $L_1 = a_1x + b_1y + c_1 = 0$ & $L_2 = a_2x + b_2y + c_2 = 0$, acute angle (θ) is given by

  $$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|,$$

  where m_1 = slope of L_1, m_2 = slope of L_2
- L_1 & L_2 are ⊥r iff $m_1m_2 = -1$
- L_1 and L_2 are parallel iff $m_1 = m_2$

Foot of Perpendicular from a Point

For a point (h, k) and line $ax + by + c = 0$

foot of ⊥r $= \frac{x - h}{a} = \frac{y - k}{b} = \frac{-(ah + bk + c)}{a^2 + b^2}$

DIFFERENTIAL EQUATIONS

Class XII

CONCEPT MAP

DIFFERENTIAL EQUATIONS

DEFINITION

An equation involving dependent variable, independent variable and derivative(s) of dependent variable(s) w.r.t. independent variable(s).

Order

It is the order of highest order derivative occurring in a differential equation.

Note : Order of a differential equation is always positive.

Degree

Degree is the power of the highest order derivative in a differential equation.

Note : (i) Degree (if defined) of a differential equation is always positive integer.

(ii) The degree of a differential equation which is not a polynomial equation in derivatives is not defined.

Formation of Differential Equation

- By eliminating arbitrary constants.
- If the order of differential equation is n, differentiate the equation n times to eliminate arbitrary constants.

SOLUTION

A function of the form $y = f(x) + c$, which satisfies the given differential equation.

Types of Solutions

Particular Solution

Obtained by assigning values to arbitrary constants.

General Solution

Contains as many arbitrary constants as order of equation.

Methods of Solving First Order and First Degree Differential Equations

Homogeneous Differential Equation

If $\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}$, where $f(x, y)$, $g(x, y)$ are homogeneous functions of the same degree in x and y, then put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$. Now, apply variable separable method.

Variable Separable

If $\frac{dy}{dx} = f(x)g(y)$, then

$$\int \frac{dy}{g(y)} = \int f(x)\,dx + c$$

Reducible to Variable Separable

If $dy/dx = f(ax + by + c)$, then put $ax + by + c = z$ and $b\frac{dy}{dx} = \frac{dz}{dx} - a$ and then apply variable separable method.

Linear Differential Equation

If $\frac{dy}{dx} + Py = Q$, where P, Q are functions of x, then $ye^{\int P dx} = \int Qe^{\int P dx} dx + c$, where $e^{\int P dx}$ is the integrating factor (I.F.).

If $\frac{dx}{dy} + Px = Q$, where P, Q are functions of y, then $xe^{\int P dy} = \int Qe^{\int P dy} dy + c$, where $e^{\int P dy}$ is the integrating factor (I.F.).

23. (a) : Given lines are mutually perpendicular and intersect at $\left(\frac{6}{5},\frac{13}{5}\right)$.

Equation of angle bisector of the given lines are

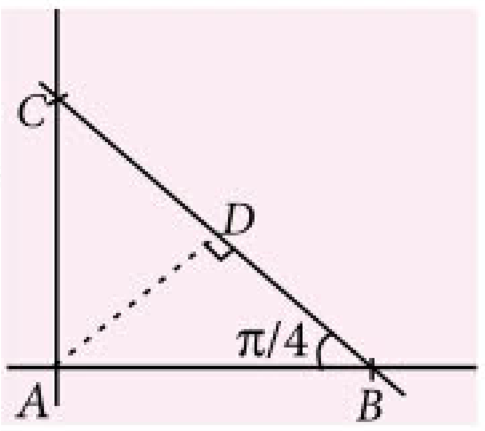
$x - 2y + 4 = \pm(2x + y - 5)$

i.e., $x + 3y = 9$, $3x - y = 1$

Side BC will be parallel to one of these bisectors.

Let $AD = a \Rightarrow AB = a\sqrt{2}$

and, area of $\Delta_{ABC} = \frac{1}{2}(a\sqrt{2})^2$



$\Rightarrow a^2 = 10 \Rightarrow a = \sqrt{10}$

Let equation of BC is $x + 3y = \lambda$

$$\Rightarrow \sqrt{10} = \frac{\left|\frac{6}{5}+\frac{39}{5}-\lambda\right|}{\sqrt{10}}$$

$\Rightarrow \lambda = -1, 19$

Thus equation of BC is

$x + 3y = -1$ or $x + 3y = 19$.

If equation of BC is $3x - y = \lambda$

$$\Rightarrow \sqrt{10} = \frac{\left|\frac{18}{5}-\frac{13}{5}-\lambda\right|}{\sqrt{10}} \Rightarrow \lambda = -9, 11$$

Hence, equation of BC is

$3x - y = -9$ or $3x - y = 11$.

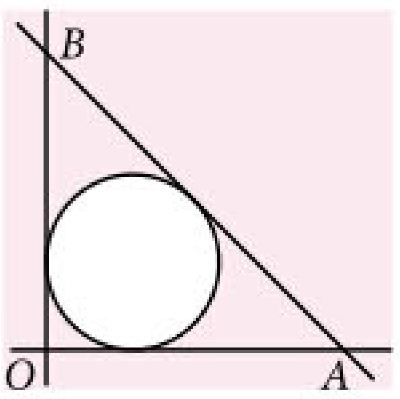
24. (b) : Let the equation of side AB be

$\frac{x}{a}+\frac{y}{b}=1$ or $bx + ay - ab = 0$

Centre and radius of circle are (2, 2) and 2 units respectively.

Since, AB touches the circles. Therefore,

$$\frac{|2b+2a-ab|}{\sqrt{a^2+b^2}} = 2$$



$\Rightarrow -(2a+2b-ab) = 2\sqrt{a^2+b^2}$

('–' sign is used as (2, 2) lies below the line $bx + ay - ab = 0$)

If $P(h, k)$ is the midpoint of AB, then $a = 2h$, $b = 2k$

Thus, $4kh - 4h - 4k = 2\sqrt{4h^2+4k^2}$

Hence, locus of 'P' is,

$$x + y - xy + \sqrt{x^2+y^2} = 0$$

$\therefore \lambda = 1$

25. (a) : Let $P \equiv (h, k)$

Tangent in terms of m is $y = mx + \frac{a}{m}$

$\Rightarrow m^2x - my + a = 0$

If it passes through (h, k), then

$m^2h - mk + a = 0$.

Since, angle between these tangents is $\pi/4$.

$$\therefore \tan\frac{\pi}{4} = \frac{|m_1 - m_2|}{|1+m_1m_2|}$$

$\Rightarrow (m_1 - m_2)^2 = (1 + m_1m_2)^2$

$\Rightarrow (m_1 + m_2)^2 - 4m_1m_2 = (1 + m_1m_2)^2$

$$\Rightarrow \frac{k^2}{h^2} - \frac{4a}{h} = \left(1+\frac{a}{h}\right)^2$$

Thus, required locus is, $y^2 = 4ax + (a + x)^2$

26. (c) : Tangent in terms of 'm' for $y^2 = 4x$ is $y = mx + \frac{1}{m}$

and for $\frac{x^2}{8}+\frac{y^2}{2}=1$ tangent is, $y = mx \pm \sqrt{8m^2+2}$

For common tangents we must have $\frac{1}{m} = \pm\sqrt{8m^2+2}$

$\Rightarrow m = \pm\frac{1}{2}$

Thus, common tangents are $y = \frac{x}{2}+2$ and $y = -\frac{x}{2}-2$

i.e. $2y - x - 4 = 0$ and $2y + x + 4 = 0$

27. (c) : Length of latus rectum $= \frac{2b^2}{a}$

For $xy = c^2$, $a = b$ and $\frac{a^2}{2} = c^2$

Required length $= 2\cdot\frac{a^2}{a} = 2a = 2\sqrt{2}c$

28. (d) : Given line is $2x = 3y = 4z$

i.e. $\frac{x}{6}=\frac{y}{4}=\frac{z}{3}$

Let the plane be $ax + by + cz = 1$

$\therefore 6a + 4b + 3c = 0$, $2a - b = 1$ and $3a - 4b + 5c = 1$

$$\Rightarrow a = \frac{29}{85},\ b = -\frac{27}{85},\ c = -\frac{22}{85}$$

Thus, equation of plane is $29x - 27y - 22z = 85$.

29. (c) : Given line passes through $P(1, 2, 4)$ and this point also lies on the given plane.

Thus, required line will be in the form of

$$\frac{x-1}{l} = \frac{y-2}{m} = \frac{z-4}{n}$$

Any point on the given line is

$(-r_1 + 1, 3r_1 + 2, r_1 + 4)$

If $r_1 = 1$, this point becomes $P \equiv (0, 5, 5)$.

Let $Q(a, b, c)$ be the reflection of 'P' in the given plane, then

$$\frac{a}{2}+\frac{b+5}{2}+\frac{5+c}{2} = 7$$

i.e., $a + b + c = 4$ and $\frac{a}{1} = \frac{b-5}{1} = \frac{c-5}{1} = \lambda$ (say)

$\Rightarrow\ a = \lambda,\ b = 5 + \lambda,\ c = 5 + \lambda$
$\Rightarrow\ 10 + 3\lambda = 4\ \Rightarrow\ \lambda = -2.$
Thus, $Q \equiv (-2, 3, 3)$
Hence, direction ratios of reflected line are $-3, 1, -1$.

Thus, its equation is $\frac{x-1}{-3} = \frac{y-2}{1} = \frac{z-4}{-1}$

30. (b) : $\frac{\sin x - \sin z}{\cos z - \cos x} = \frac{2\cos\left(\frac{x+z}{2}\right).\sin\left(\frac{x-z}{2}\right)}{2\sin\left(\frac{x+z}{2}\right).\sin\left(\frac{x-z}{2}\right)}$

$= \cot\left(\frac{x+z}{2}\right) = \cot(y)$

31. (c) : We have, $27^{\cos x} + 81^{\sin x}$

$= 3^{3\cos x} + 3^{4\sin x} \geq 2\cdot\sqrt{3^{3\cos x}\cdot 3^{4\sin x}}$

$= 2\cdot 3^{\frac{1}{2}(3\cos x + 4\sin x)} \geq 2\cdot 3^{-\frac{5}{2}} = \frac{2}{9\sqrt{3}}$

32. (c) : $\sin^{-1}\left(\frac{\sqrt{r}-\sqrt{r-1}}{\sqrt{r(r+1)}}\right) = \tan^{-1}\left(\frac{\sqrt{r}-\sqrt{r-1}}{1+\sqrt{r(r-1)}}\right)$

$= \tan^{-1}\sqrt{r} - \tan^{-1}(\sqrt{r-1})$

$\Rightarrow\ \sum_{r=1}^{n}\sin^{-1}\left(\frac{\sqrt{r}-\sqrt{r-1}}{\sqrt{r(r+1)}}\right) = \sum_{r=1}^{n}\left(\tan^{-1}\sqrt{r} - \tan^{-1}\sqrt{r-1}\right)$

$= \tan^{-1}\sqrt{n}$

33. (b) : Clearly, the direction ratios of normal to the plane are 4, –3, –2
Thus equation of plane will be $4x - 3y - 2z = d$
It will necessarily pass through $(3, -2, -1)$
i.e., $d = 12 + 6 + 2 = 20$
Thus the equation of plane is $4x - 3y - 2z = 20$.

34. (d) : $\angle A = \pi/2\ \Rightarrow\ 2R = a$

Also, $r = (s - a)\tan\frac{A}{2} = (s-a) = \frac{1}{2}(b+c-a)$

$\Rightarrow\ r + 2R = \frac{1}{2}(a + b + c)$

35. (b) : Let Clearly, $a + b > c$

$\Rightarrow\ (\sqrt{a}+\sqrt{b})^2 - 2\sqrt{ab} > (\sqrt{c})^2$

$\Rightarrow\ (\sqrt{a}+\sqrt{b})^2 > (\sqrt{c})^2\ \Rightarrow\ \sqrt{a}+\sqrt{b} > \sqrt{c}$

$\Rightarrow\ \sqrt{a}+\sqrt{b}-\sqrt{c} > 0$

36. (a) : Let $I = \int \tan^{-1}\sqrt{x}\,dx$

$= x\cdot\tan^{-1}\sqrt{x} - \int\frac{1}{1+x}.\frac{1}{2\sqrt{x}}.x\,dx$

$= x\cdot\tan^{-1}\sqrt{x} - \frac{1}{2}\int\frac{\sqrt{x}}{1+x}dx$

Now, putting $\sqrt{x} = t$ *i.e.,* $\frac{1}{2\sqrt{x}}dx = dt$

$\Rightarrow\ dx = 2t\,dt$, we get

$I = x\cdot\tan^{-1}\sqrt{x} - \frac{1}{2}\int\frac{t}{1+t^2}2t\,dt$

$= x\tan^{-1}\sqrt{x} - \int\frac{t^2+1-1}{1+t^2}dt$

$= x\tan^{-1}\sqrt{x} - t + \tan^{-1}t + C$

$= x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x} + C$

37. (d) : Clearly, the given curves intersect for $a \in R - \{0\}$

For the parabola, $\frac{dy}{dx} = \frac{8}{y}$. For the ellipse, $\frac{dy}{dx} = \frac{-4x}{a^2y}$

For the curves to intersect orthogonally,

$\left(\frac{8}{y}\right)\left(-\frac{4x}{a^2y}\right) = -1$

$\Rightarrow\ a^2y^2 = 32x\ \Rightarrow\ 16a^2x = 32x \Rightarrow a^2 = 2$

38. (c) : We have, $f'(x) = 2e^x + ae^{-x} + 2a + 1$

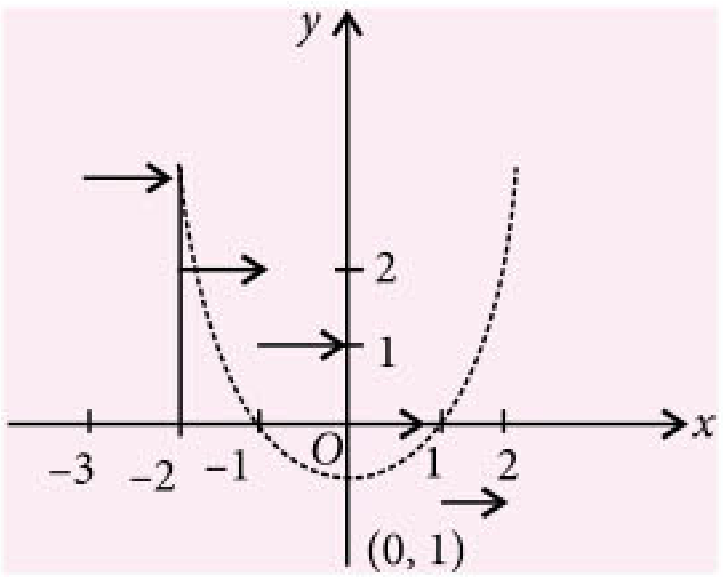
$= e^{-x}(2e^{2x} + (2a+1)e^x + a)$

$= 2e^{-x}\left(e^{2x} + \left(a+\frac{1}{2}\right)e^x + \frac{a}{2}\right)$

$= 2e^{-x}(e^x + a)\left(e^x + \frac{1}{2}\right)$

For $f(x)$ to be increasing, $f'(x) \geq 0\ \forall\ x \in R$
$\Rightarrow\ e^x + a \geq 0\ \forall\ x \in R\ \Rightarrow a \geq 0$

39. (d) : We must have $[x] - 1 + x^2 \geq 0$
$\Rightarrow\ x^2 - 1 \geq -[x]$
The graphs of $y = x^2 - 1$ and $y = -[x]$ intersect somewhere in $(-2, -1)$. Thus at the point of intersection, $x^2 - 1 = 2$
$\Rightarrow\ x = -\sqrt{3}$
From the adjacent graph we get domain as $(-\infty, -\sqrt{3}] \cup [1, \infty)$



40. (a) : $\frac{r}{(r+1)!} = \frac{r+1-1}{(r+1)!} = \frac{1}{r!} - \frac{1}{(r+1)!}$

$= -\left(\frac{1}{(r+1)!} - \frac{1}{r!}\right) = -(V(r) - V(r-1))$

$\sum_{r=1}^{n}\frac{r}{(r+1)!} = -(V(n) - V(0)) = 1 - \frac{1}{(n+1)!}$

◆

JEE MAIN

Exam Dates
OFFLINE : 8th April
ONLINE : 15th & 16th April

PRACTICE PAPER 2018

1. The polynomial $4x^4 - ax^3 + bx^2 - cx + 5 = 0$ has four positive real roots (r_1, r_2, r_3, r_4), $(r_1 < r_2 < r_3 < r_4)$ such that $\frac{r_1}{2}+\frac{r_2}{4}+\frac{r_3}{5}+\frac{r_4}{8}=1$ then $2r_1 + r_2 + 4r_3 + r_4 =$

(a) 8 (b) 9 (c) 10 (d) 12

2. Let $x, y \in \left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$ and $x^3 + \sin x = 2a$,

$4y^3 + \frac{\sin 2y}{2} + a = 0$ then $\cos(x + 2y) =$

(a) 0 (b) 1 (c) –1 (d) $\frac{1}{\sqrt{2}}$

3. For positive x, the min. value of

$x^{1000} + x^{900} + x^{90} + x^6 + \frac{1996}{x}$ is

(a) 1996 (b) 1998
(c) 2000 (d) 2002

4. The number of values of a for which $\frac{x^3 - 6x^2 + 11x - 6}{x^3 + x^2 - 10x + 8} + \frac{a}{30} = 0$ does not have a real solution is

(a) 1 (b) 2 (c) 3 (d) 4

5. If for some natural number n, $(1 + 2 + 3 + ... + n) + k = 2013$, where k is one of the numbers 1, 2, 3, ..., n then $n + k =$

(a) 120 (b) 122 (c) 124 (d) 134

6. If A and B are two square matrices such that $A + B + AB = O$ then $AB =$

(a) BA (b) $-BA$ (c) A (d) B

7. The number of values of n such that $\log_n(2^{231})$ is an integer is $(n \in N)$

(a) 6 (b) 7 (c) 8 (d) 9

8. The plane $lx + my + nz = p$ intersects co-ordinate axes in A, B and C. If area of ΔABC is Δ, then

(a) $p^2 \le \frac{2\Delta}{3\sqrt{3}}$ (b) $p^2 \ge \frac{2\Delta}{3\sqrt{3}}$

(c) $p^2 < \frac{2\Delta}{3\sqrt{3}}$ (d) $p^2 > \frac{2\Delta}{3\sqrt{3}}$

9. If k is a positive integer such that $36 + k$, $300 + k$, $596 + k$ are the squares of three consecutive terms of an A.P., then number of such possible progressions are

(a) 0 (b) 1 (c) 2 (d) 3

10. In ΔABC, $\sqrt{3}\sin C = 2\sec A - \tan A$ and one of the sides has lengths 2 units then maximum area of ΔABC is

(a) 2 (b) $2\sqrt{2}$ (c) $2\sqrt{3}$ (d) 4

11. If circles $x^2 + y^2 = c$, (radius $= \sqrt{3}$) and $x^2 + y^2 + ax + by + c = 0$, (radius $= \sqrt{6}$) intersect at two points P and Q, then length of chord PQ is

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $\sqrt{3}$ (d) $2\sqrt{3}$

12. A rod of fixed length t has its ends sliding along the coordinate axes in 1st quadrant such that its end points are at $A(a, 0)$ and $B(0, b)$, then min. value of $\left(a+\frac{1}{b}\right)^2 + \left(b+\frac{1}{a}\right)^2$ is

(a) $\left(t+\frac{2}{t}\right)^2$ (b) $\left(1+\frac{1}{t}\right)^2$

(c) $\left(t+\frac{1}{t}\right)^2$ (d) $\left(1+\frac{2}{t}\right)^2$

13. Two tangents $x - 2y + 10 = 0$ and $x = 2$ of a parabola intersect the tangent at vertex at (–2, 4) and (2, 0) respectively, then length of latus rectum of the parabola is

By : Tapas Kr. Yogi, Mob : 9533632105.

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $3\sqrt{2}$ (d) $4\sqrt{2}$

14. If lines $x + y = 3$ and $y = x + 1$ are two tangents to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then angle between pair of tangents drawn from $(4, -\sqrt{5})$ to $\frac{x^2}{3a^2+b^2}+\frac{y^2}{a^2+3b^2}=1$ can be

(a) $\pi/3$ (b) $2\pi/3$ (c) $3\pi/4$ (d) $5\pi/6$

15. The number of integral values of x for which $\sin^{-1}\left(\frac{4x}{x^2+4}\right)-2\tan^{-1}\left(\frac{x}{2}\right)$ is independent of x is

(a) 3 (b) 4 (c) 5 (d) 8

16. The value of ${}^{40}C_0 \cdot {}^{100}C_{40} - {}^{40}C_1 \cdot {}^{99}C_{40} + {}^{40}C_2 \cdot {}^{98}C_{40} - .... + {}^{40}C_{40} \cdot {}^{60}C_{40}$ is

(a) ${}^{40}C_{40}$ (b) ${}^{100}C_{40}$ (c) ${}^{140}C_{40}$ (d) ${}^{140}C_{80}$

17. A random variable x follows a binomial distribution with $n = 10$ and $p = 0.4$. What value of x is most likely to be obtained?

(a) 2 (b) 3 (c) 4 (d) 5

18. Which of the following statement is a tautology?

(a) $((p \to q) \wedge (\sim q)) \to (\sim p)$
(b) $((p \to q) \wedge (\sim q)) \to p$
(c) $((p \to q) \wedge q) \to (\sim p)$
(d) $((p \to q) \wedge q) \to p$

19. The value of $\sum_{r=1}^{n+1}\left(\sum_{k=1}^{n} {}^{k}C_{r-1}\right), (n, r, k \in N)$ is

(a) 2^{n+1} (b) $2^{n+1} - 1$
(c) $2^{n+1} - 2$ (d) 2^n

20. If $|x - y| + |x + y| = 2$ then max. possible value of $x^2 - 6x + y^2$ is

(a) 7 (b) 8 (c) 9 (d) 10

21. There are real numbers a and b such that for every positive x, we have the identity

$\tan^{-1}\left(\frac{1}{x}-\frac{x}{8}\right)+\tan^{-1}(ax)+\tan^{-1}(bx)=\frac{\pi}{2}$, then $a^2+b^2=$

(a) 1/4 (b) 2/4 (c) 3/4 (d) 4/4

22. Let f and g be two differentiable functions on R satisfying $f(x)=\frac{x^3}{2}+1-x\int_0^x g(t)dt$ and $g(x)=x-\int_0^1 f(t)\,dt$. The number of points where $f(|x|)$ is not differentiable is

(a) 0 (b) 1 (c) 2 (d) 3

23. The domain of the function $f(x) = \log(-a-|1+[x]|)$ where a is fixed negative real, $a \notin I$, is

(a) $[a] \le x < [-a]$ (b) $[a] \le x \le [-a]$
(c) $[a] + 1 \le x \le [-a]$ (d) $[a] + 1 \le x < 1 + [-a]$

24. Let $f(x)=(x+1)^2+\frac{1}{x}(x+1)$, then

$$\int_{-2}^{1} f(x)\cdot f(-x)dx =$$

(a) 2 (b) $\frac{-9}{10}$
(c) $\frac{9}{10}$ (d) does not exist

25. Which of the following relations R in the set Z of integers is not an equivalence relation?

(a) $R = \{(a, b)\ ;\ 3 \text{ divides } (a - b)\}$
(b) $R = \{(a, b)\ ;\ 2 \text{ divides } (a - b)\}$
(c) $R = \{(a, b)\ ;\ 2 \text{ divides } (a + b)\}$
(d) $R = \{(a, b)\ ;\ 3 \text{ divides } (a + b)\}$

26. Let $f(x)$ be such that

$(1-x)\cdot 2^{-f(x)}=(x+1)(x+\sqrt{x^2+1})$ then

(a) the graph of $y = f(x)$ is symmetric about $y = x$
(b) $f(0) = 1$ (c) $f'(0) = 0$
(d) $f''(0) = 0$

27. Let $f : R \to R$ be a continuous function such that $f(x) \cdot f(f(x)) = 1\ \forall\ x \in R$. Let $f(10) = 9$, then

(a) $f(1) \neq 1$ (b) $f(19)=\frac{1}{19}$
(c) $f(5)=\frac{1}{5}$ (d) $f\left(\frac{11}{100}\right)=\frac{100}{11}$

28. Two lines $5x + 3^t y = 1$ and $tx - y = 0$ are perpendicular to each other for some value of $t \in$

(a) $(-1, 0)$ (b) $(1, 2)$
(c) $(2, 3)$ (d) $(3, 4)$

29. The function $f(x) = a(x^2 - 1)(ax + b), a \neq 0$ has

(a) a point of local maxima at certain $x_0 \in R^+$
(b) a point of local minima at certain $x_0 \in R^-$
(c) a point of local maxima at certain $x_0 \in R^-$
(d) no point of local maxima or minima

30. The minimum value of $(a^2x^2 + b^2y^2)$ under the condition that $\frac{a^2}{x^2}+\frac{b^2}{y^2}=1$ is

(a) $(a^2 + b^2)^2$ (b) $(a^2 - b^2)^2$
(c) $a^2b^2(a^2 + b^2)^2$ (d) $a^4 + b^4$

SOLUTIONS

1. (b) : $4x^4 - ax^3 + bx^2 - cx + 5 = 0$, $r_1r_2r_3r_4 = \frac{5}{4}$
and AM-GM in equation gives

$$\frac{r_1}{2}+\frac{r_2}{4}+\frac{r_3}{5}+\frac{r_4}{8} \ge 4\cdot\sqrt[4]{\frac{r_1}{2}\cdot\frac{r_2}{4}\cdot\frac{r_3}{5}\cdot\frac{r_4}{8}}$$

i.e. $1 \ge 4\cdot\sqrt[4]{\frac{r_1r_2r_3r_4}{2\cdot4\cdot5\cdot8}} \Rightarrow r_1r_2r_3r_4 \le \frac{5}{4}$

i.e. equality is obtained when $\frac{r_1}{2}=\frac{r_2}{4}=\frac{r_3}{5}=\frac{r_4}{8}$

Hence, the roots are $r_1 = \frac{1}{2}, r_2 = 1,\ r_3 = \frac{5}{4}$ and $r_4 = 2$.

2. (b) : Let $z = 2y$, then 2nd equation becomes

$z^3 + \sin z + 2a = 0$ *i.e.* $x^3 + \sin x = -z^3 - \sin z$

Let $f(x) = x^3 + \sin x$, then $f(x)$ is odd and strictly increasing in $[-\pi/4, \pi/4]$. So $f(x) = f(-z)$
$\Rightarrow x = -z = -2y$
Hence, $\cos(x + 2y) = \cos 0 = 1$

3. (c) : $\dfrac{x^{1000}+x^{900}+x^{90}+x^{6}+\frac{1}{x}+\ldots+\frac{1}{x}}{2000} \ge 1$

Hence, required expression ≥ 2000.

4. (c) : $\dfrac{x^3-6x^2+11x-6}{x^3+x^2-10x+8} = \dfrac{(x-1)(x-2)(x-3)}{(x-1)(x-2)(x+4)}$

domain = $R - \{1, 2, -4\}$

So, range = $R - \left\{1, \frac{-2}{5}, \frac{-1}{6}\right\}$

So, equation will have no real solution if

$$\frac{a}{30} = -1,\ \frac{2}{5},\ \frac{1}{6}$$

5. (b) : $\dfrac{n(n+1)}{2} + k = 2013$ and $n > k$

i.e. $n > 2013 - \dfrac{n(n+1)}{2}$

Solving, $n \in (61.9, 62.9)$
So, $n = 62$ (integral) and $k = 60$

6. (a) : $AB + A + B = O \Rightarrow A + B + AB + I = I$
So, $(A + I)(B + I) = I$
i.e. $(A + I)$ and $(B + I)$ are inverse of each other.
Hence, $(A + I)(B + I) = I = (B + I)(A + I)$
i.e. $AB = BA$

7. (c) : $\log_n(2^{231}) = 3\times7\times11\cdot\log_n(2) = 3\cdot7\cdot11\cdot\dfrac{\log 2}{\log_n}$

So, for this expression to be integer, possible values of n are 8 in number
$(2, 2^3, 2^7, 2^{11}, 2^{3\times7}, 2^{3\times11}, 2^{7\times11}$ and $2^{3\times7\times11})$

8. (a) : $\Delta = \sqrt{\Delta_x^2+\Delta_y^2+\Delta_z^2}$

$$= \sqrt{\frac{1}{4}\left[\left(\frac{p^2}{lm}\right)^2+\left(\frac{p^2}{mn}\right)^2+\left(\frac{p^2}{ln}\right)^2\right]}$$

i.e., $\Delta = \dfrac{p^2}{2lmn}$

AM-GM inequality gives $\dfrac{l^2+m^2+n^2}{3} \ge (lmn)^{2/3}$

i.e., $\dfrac{1}{3\sqrt{3}} \ge \dfrac{p^2}{2\Delta}$

Equality occurs at $l = m = n = \dfrac{1}{\sqrt{3}}$

9. (c) : We have, $36 + k = (a - d)^2$, $300 + k = a^2$ and $596 + k = (a + d)^2$
Solving, we have $d = \pm 4$

10. (c) : Simplifying the given expression, we have

$\sqrt{3}\cos A\sin C = 2 - \sin A$

$\Rightarrow$ $\cos A$ is +ve *i.e.* $\angle A$ is acute
and $\sin C \le 1 \Rightarrow 2 - \sin A \le \sqrt{3}\cos A$
i.e., $\sqrt{3}\cos A + \sin A \ge 2 \Rightarrow \sqrt{3}\cos A + \sin A = 2$
$\Rightarrow$ $\sin C = 1$ or $\angle C = 90°$ and $A = 30°$, $B = 60°$
For maximum area, side a or b should be 2 units.

So, $\Delta_{\max} = \frac{1}{2}\cdot2\cdot2\sqrt{3} = 2\sqrt{3}$ sq. units

11. (b) : Notice that the given circles are orthogonal.

Hence, PQ length = $\dfrac{2r_1r_2}{\sqrt{r_1^2+r_2^2}} = 2\sqrt{2}$

12. (a) : $a^2 + b^2 = t^2$ and

$$\left(a+\frac{1}{b}\right)^2+\left(b+\frac{1}{a}\right)^2 = a^2+b^2+\frac{1}{a^2}+\frac{1}{b^2}+2\left(\frac{a}{b}+\frac{b}{a}\right)$$

AM-HM inequality gives $\dfrac{1}{a^2}+\dfrac{1}{b^2} \ge \dfrac{4}{t^2}$

Hence, given expression $\ge t^2 + \dfrac{4}{t^2} + 4 = \left(t+\dfrac{2}{t}\right)^2$

13. (d) : Focus is point of intersection of lines perpendicular to lines $x - 2y + 10 = 0$ and $x = 2$ and passing through $(-2, 4)$ and $(2, 0)$ respectively.
Hence, focus = (0, 0).

14. (a) : Notice that tangents are perpendicular, so their point of intersection (1, 2) should lie on director circle $x^2 + y^2 = a^2 + b^2 \Rightarrow a^2 + b^2 = 5$.

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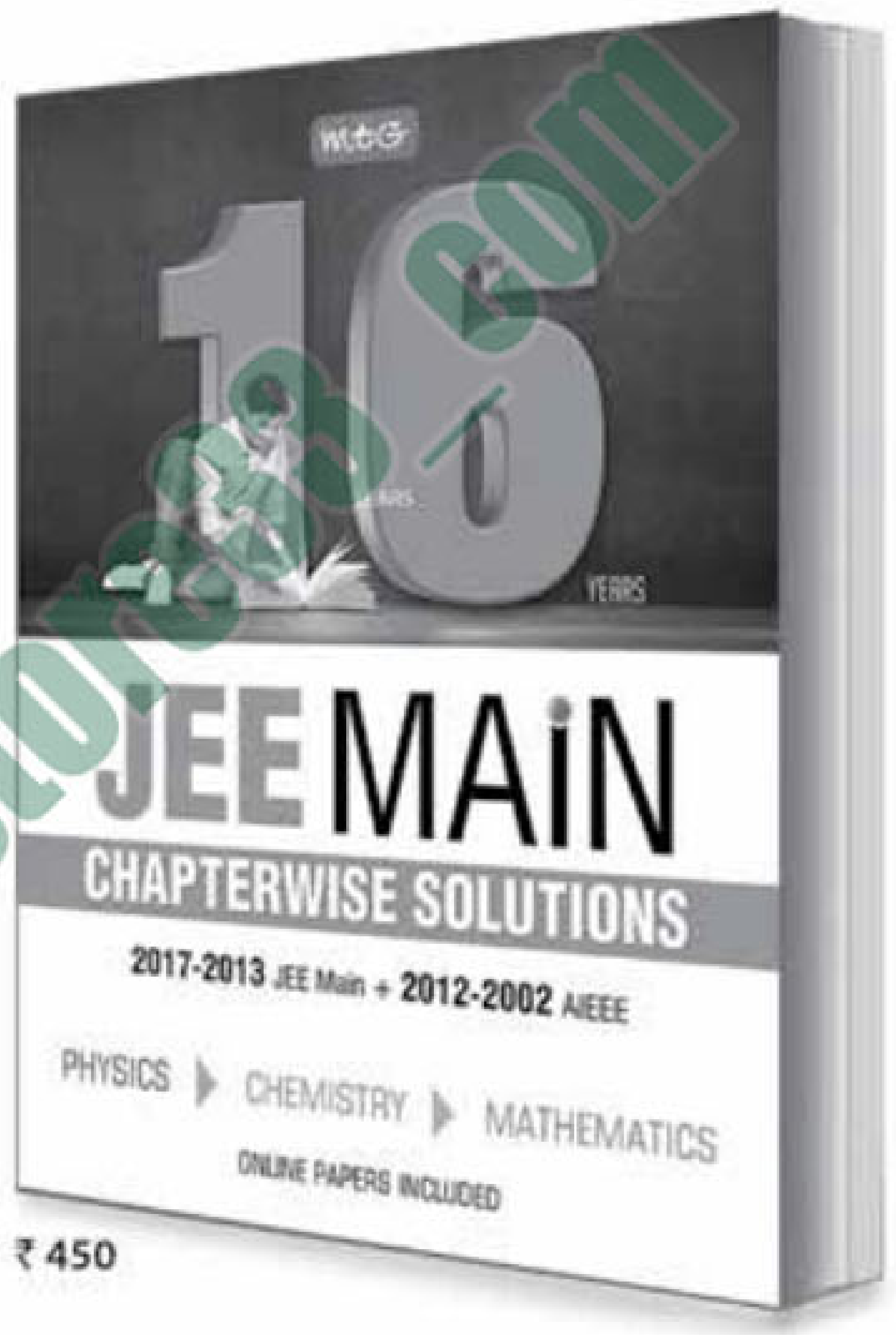
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Now, director circle of 2nd ellipse is

$x^2 + y^2 = 4(a^2 + b^2)$ *i.e.*, $x^2 + y^2 = 20$.

$(4, -\sqrt{5})$ lies outside this circle.

Hence, angle between the tangents is acute.

15. (c) : We have, $\sin^{-1}\left(\frac{2\cdot(x/2)}{1+(x/2)^2}\right)-2\tan^{-1}\left(\frac{x}{2}\right)$

$$\Rightarrow\ 2\tan^{-1}\left(\frac{x}{2}\right)-2\tan^{-1}\left(\frac{x}{2}\right)=0$$

provided, $\left|\frac{x}{2}\right|\le 1\ \Rightarrow\ x\in[-2,2]$

16. (a) : The given sum is coefficient of x^{40} in
${}^{40}C_0(1 + x)^{100} - {}^{40}C_1(1 + x)^{99} + \ldots$
$+ (-1)^{40}\cdot{}^{40}C_{40}(1 + x)^{60} = 1$

17. (c) : If $p(x)$ is more likely, then $p(x) \ge p(x + 1)$
${}^{10}C_x\cdot 4^x\cdot 6^{10-x} \ge {}^{10}C_{x+1}\cdot 4^{x+1}\cdot 6^{10-(x+1)}$

Solving, $x\ge\frac{17}{5}$. Hence, $x = 4$

18. (a) : Constructing the truth table, we get only in option (a), the whole column has T in it.

19. (c) : Given : sum $=\sum_{r=1}^{n+1}\left(\sum_{k=1}^{n}({}^{k+1}C_r-{}^{k}C_r)\right)$

$$=\sum_{r=1}^{n+1}({}^{n+1}C_r-{}^{1}C_r)=2^{n+1}-2$$

20. (b) : $|x - y| + |x + y| = 2$ represents a square with vertices at $(\pm1, \pm1)$ and $x^2 - 6x + y^2 = (x - 3)^2 + y^2 - 9$ is the distance of any point on the square from (3, 0).

21. (c) : Let $A=\tan^{-1}\left(\frac{8-x^2}{8x}\right)$, $B=\tan^{-1}(ax)$,
$C = \tan^{-1}(bx)$ then given

$A + B + C = \pi/2 \ \Rightarrow\ \Sigma\tan A\tan B = 1$

So, on simplifying we have,

$$(a+b)\left(1-\frac{x^2}{8}\right)+abx^2=1$$

i.e. $x^2\left(ab-\frac{a}{8}-\frac{b}{8}\right)+a+b=1$

For this to be an identity, we should have,

$$ab-\frac{a}{8}-\frac{b}{8}=0 \ \text{ and } a + b = 1$$

Hence, $a^2+b^2=\frac{3}{4}$

22. (a) : Let $a=\int_0^1 f(t)\,dt$, then $g(x) = x - a$ and

$f(x) = ax^2 + 1$.

So, $a=\int_0^1(at^2+1)\,dt$ gives $a = 3/2$,

i.e. $f(x)=\frac{3}{2}x^2+1$

23. (a) : $|1 + [x]| < -a \ \Rightarrow\ a - 1 < [x] < -a - 1$
$\Rightarrow\ [a] \le x < [-a]$

24. (d) : $f(x)\cdot f(-x)=x^4-\frac{1}{x^2}$

So, $\int_{-2}^{1} f(x)f(-x)dx=\int_{-2}^{0} f(x)f(-x)dx+\int_{0}^{1} f(x)f(-x)dx$

$$=\left.\frac{x^5}{5}+\frac{1}{x}\right|_{-2}^{0^-}+\left.\frac{x^5}{5}+\frac{1}{x}\right|_{0^+}^{1}$$

$\therefore$ Value does not exist.

25. (d) : $(a + a) = 2a$ may not be a multiple of 3.
So, option (d) is not reflexive.

26. (d) : $f(x)=\log_2\left(\frac{1-x}{1+x}\right)-\log_2(x+\sqrt{x^2+1})$

$f(x)$ is an odd function. So, $f''(0) = 0$.

27. (c) : $f(10) = 9 \ \Rightarrow\ 9f(9) = 1$ *i.e.* $f(9)=\frac{1}{9}$

So, $\left[\frac{1}{9},\ 9\right]\subseteq$ Range

Hence, $f(5)=\frac{1}{5}$ possible.

28. (c) : The lines are perpendicular. So, $5t - 3^t = 0$.
Let $f(t) = 5t - 3^t$ then $f(2)\cdot f(3) < 0$
So, atleast one value of t lies in (2, 3).

29. (c) : $f(x) = a^2x^3 + abx^2 - a^2x - ab$
$f'(x) = 3a^2x^2 + 2bax - a^2$, product of roots is –ve.
So, $f(x)$ has a local maxima at some $x_0 \in R^-$ and a local minima at some $x_0 \in R^+$.

30. (a) : Let $x = a\sec\theta$, $y = b\operatorname{cosec}\theta$, then
$a^2x^2 + b^2y^2 = a^4 + b^4 + a^4\tan^2\theta + b^4\cot^2\theta$
$\ge a^4 + b^4 + 2a^2b^2 = (a^2 + b^2)^2$

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MATH archives

10 BEST PROBLEMS
$(x+y)^2 = x^2$

Math Archives, as the title itself suggests, is a collection of various challenging problems related to the topics of JEE (Main cum Advanced) Syllabus. This section is basically aimed at providing an extra insight and knowledge to the candidates preparing for JEE (Main cum Advanced). In every issue of MT, challenging problems are offered with detailed solution. The reader's comments and suggestions regarding the problems and solutions offered are always welcome.

1. In a sequence of 21 terms, the first 11 terms are in A.P. with common difference 2 and the last 11 terms are in G.P. with common ratio 2. If the middle term of the A.P. is equal to the middle term of the G.P., then the middle term of the entire sequence is

(a) $-10/31$ (b) $10/31$ (c) $2/31$ (d) $31/32$

2. If the roots of the equation $ax^2+bx+c=0$ are real and distinct, then

(a) both roots are greater than $-b/2a$
(b) both roots are less than $-b/2a$
(c) one of the roots exceeds $-b/2a$
(d) one of the roots must be negative

3. If $z_1 \neq -z_2$ and $|z_1+z_2| = \left|\frac{1}{z_1}+\frac{1}{z_2}\right|$ then $(z_1, z_2 \in C)$

(a) at least one of z_1, z_2 is unimodular
(b) both z_1, z_2 are unimodular
(c) z_1z_2 is unimodular
(d) $z_1 + z_2$ is unimodular

4. The number of real negative terms in the binomial expansion of $(1+ix)^{4n-2}, n \in N, x > 0$ is

(a) n (b) $n+1$ (c) $n-1$ (d) $2n$

5. Number of ways in which 5 balls can be selected from a bag containing 5 identical and 5 different balls is

(a) 24 (b) 30 (c) 32 (d) 36

6. The statement $\sim P \wedge (\sim Q \wedge R) \vee (Q \wedge R) \vee (P \wedge R)$ is

(a) $\Leftrightarrow P$ (b) $\Leftrightarrow R$ (c) $\Leftrightarrow Q$ (d) $\Leftrightarrow \sim R$

7. If A and B are square matrices such that $A^{2006} = O$ and $AB = A + B$, then det (B) is

(a) 0 (b) 1 (c) 2 (d) 3

8. Let $\vec{a}$ and $\vec{b}$ be unit vectors that are perpendicular to each other, then $[\vec{a}+(\vec{a}\times\vec{b})\ \ \vec{b}+(\vec{a}\times\vec{b})\ \ \vec{a}\times\vec{b}]$ will always be equal to

(a) 1 (b) zero (c) -1 (d) 3

9. Four dice are rolled. The number of possible outcomes in which at least one die shows 2 is

(a) 296 (b) 25 (c) 671 (d) 625

10. If x takes negative permissible value, then $\sin^{-1}x$ is

(a) $\cos^{-1}\sqrt{1-x^2}$ (b) $-\cos^{-1}\sqrt{1-x^2}$
(c) $\cos^{-1}\sqrt{x^2-1}$ (d) $\pi-\cos^{-1}\sqrt{1-x^2}$

SOLUTIONS

1. (a) : Given, $a+10=(a+20)2^5 \Rightarrow -31a=630$

$a = -630/31$

$$\text{Now, } a+20=\frac{-630}{31}+20=-\frac{10}{31}$$

2. (c) : The roots of the given equation are

$$\alpha=\frac{-b-\sqrt{b^2-4ac}}{2a} \text{ and } \beta=\frac{-b+\sqrt{b^2-4ac}}{2a}$$

Since α, β are real and distinct, therefore $b^2-4ac>0$.

Now, if $a>0$, then $\beta > \frac{-b}{2a}$

and if $a<0$ then $\alpha > -\frac{b}{2a}$

Thus, one of the roots exceeds $-\frac{b}{2a}$

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3. (c) : $|z_1+z_2| = \left|\dfrac{z_1+z_2}{z_1z_2}\right|$

$\Rightarrow\ |z_1+z_2|\left(1-\dfrac{1}{|z_1z_2|}\right)=0\ \Rightarrow |z_1z_2|=1$

4. (a) : $T_{r+1} = {}^{4n-2}C_r(ix)^r$

T_{r+1} is negative, if i^r is negative and real (since $x > 0$)

$i^r = -1 \Rightarrow\ r = 2, 6, 10, \ldots,$ which forms an A.P.

Now, $0 \le r \le 4n - 2$

Also, $4n - 2 = 2 + (r - 1)4 \Rightarrow r = n$

$\therefore$ Required number of terms is n.

5. (c) : Let the balls be $a, a, a, a, a, b_1, b_2, b_3, b_4, b_5$

Number of ways of selecting r identical balls is one and number of ways of selecting $5 - r$ balls from 5 different balls is $\dbinom{5}{5-r}=\dbinom{5}{r}$

$\therefore\ \displaystyle\sum_{r=0}^{5}\binom{5}{r}=2^5=32$

6. (b) : $\sim P\wedge(\sim Q\wedge R)\vee((Q\vee P)\wedge R)$

$\Leftrightarrow ((\sim P\wedge \sim Q)\wedge R)\vee((Q\vee P)\wedge\ R)$

$\Leftrightarrow\ ((\sim P \wedge \sim Q)\ \vee\ (Q\ \vee\ P))\wedge\ R$

$\Leftrightarrow\ (\sim(P\vee Q)\ \vee\ (Q\ \vee\ P))\ \wedge\ R$

($\because \sim P \vee P$ always true).

$\Leftrightarrow\ T\wedge R$

$\Leftrightarrow\ R$

7. (a) : $AB = A + B \Rightarrow\ B = AB - A = A\ (B - I)$

$\Rightarrow\ \det(B) = \det(A)\ \det(B - I) = 0 \Rightarrow \det(B) = 0$

8. (a) : $[\vec a+(\vec a\times\vec b)\ \ \vec b+(\vec a\times\vec b)\ \ \vec a\times\vec b]$

$=(\vec a+(\vec a\times\vec b)).((\vec b+(\vec a\times\vec b))\times(\vec a\times\vec b))$

$=(\vec a+(\vec a\times\vec b))\cdot(\vec b\times(\vec a\times\vec b))=\vec a\cdot\vec a+\vec a\cdot(\vec a\times\vec b)=1$

(as $\vec a\cdot\vec b=0, \vec a\cdot(\vec a\times\vec b)=0$)

9. (c) : The total number of possible outcomes is 6^4. The number of possible outcomes in which 2 does not appear on any die is 5^4. Therefore, the number of possible outcomes in which at least one die shows 2 is $6^4 - 5^4 = 1296 - 625 = 671$

10. (b) : If $x < 0$ then $\sin^{-1}x < 0$ but $\cos^{-1}\sqrt{1-x^2}$ is always positive

Then $\sin^{-1}x=-\cos^{-1}\sqrt{1-x^2}$

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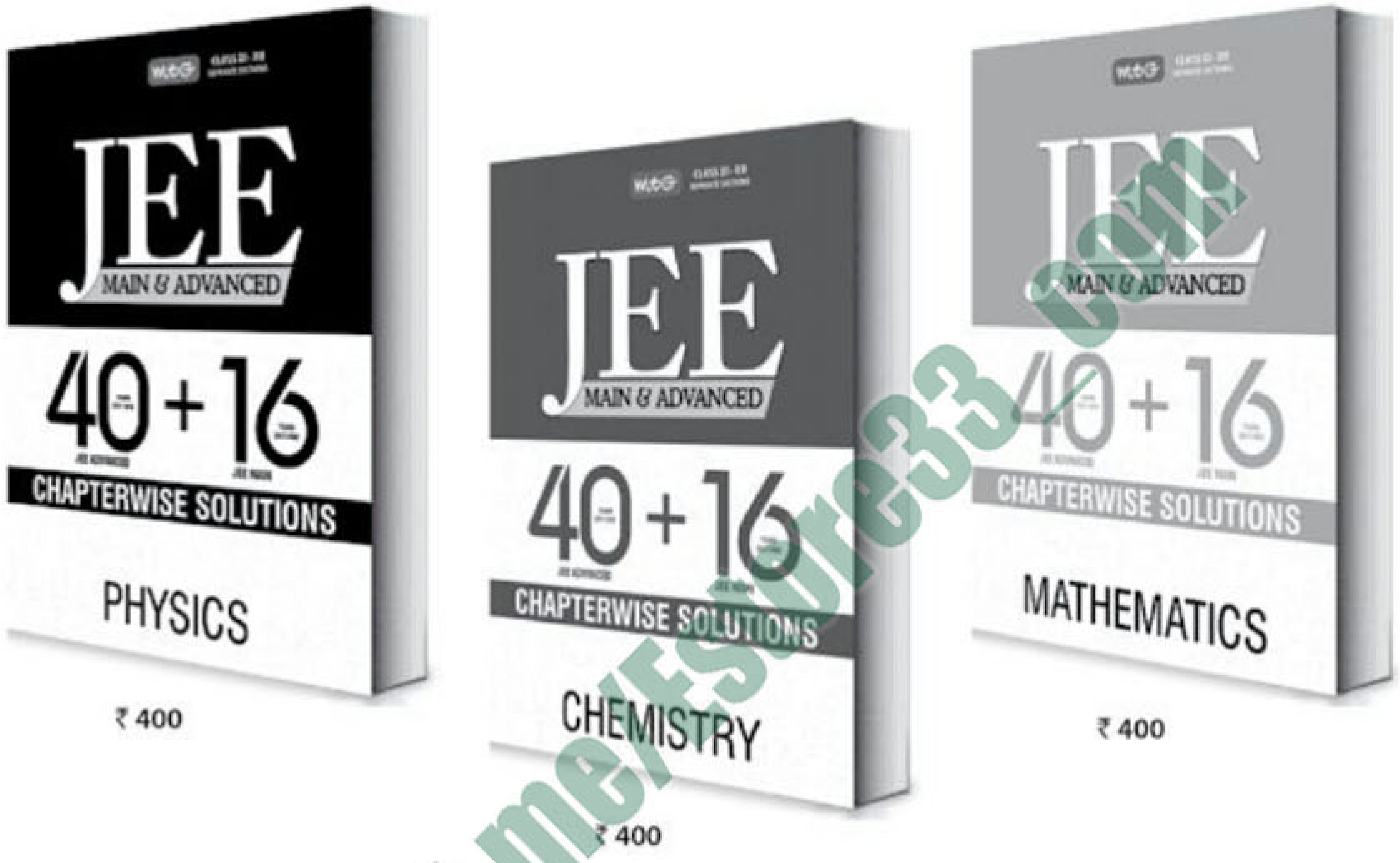

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MPP MONTHLY Practice Problems

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer.

Self check table given at the end will help you to check your readiness.

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. There are three piles of identical red, blue and green balls and each pile contains at least 10 balls. The number of ways of selecting 10 balls if twice as many red balls as green balls are to be selected is

(a) 5 (b) 4 (c) 3 (d) 2

2. If the slopes of the lines $3x^2 + 2hxy + 4y^2 = 0$ are in the ratio 3 : 1, then h equals

(a) $\frac{1}{4}$ (b) ± 4 (c) $-\frac{1}{2}$ (d) $-\frac{1}{4}$

3. If $\sin^4 x + \cos^4 y + 2 = 4 \sin x \cos y$ and $0 \le x, y \le \frac{\pi}{2}$, then $\sin x + \cos y$ is equal to

(a) -2 (b) 0 (c) 2 (d) $\frac{3}{2}$

4. The coefficient of x^{n-2} in the polynomial $(x-1)(x-2)(x-3) \dots (x-n)$ is

(a) $\frac{n(n^2+2)(3n+1)}{24}$ (b) $\frac{n(n^2-1)(3n+2)}{24}$

(c) $\frac{n(n^2+1)(3n+4)}{24}$ (d) none of these

5. Different words are being formed by arranging the letters of the word "SUCCESS". All the words obtained by writing in the form of a dictionary. The number of words in which the two C are together but no two S are together, is

(a) 120 (b) 96 (c) 24 (d) 420

6. If the sides of a right angled triangle form an A.P., then the sines of the acute angles are

(a) $\frac{3}{5}, \frac{4}{5}$ (b) $\sqrt{3}, \frac{1}{3}$

(c) $\sqrt{\left(\frac{\sqrt{5}-1}{2}\right)}, \sqrt{\left(\frac{\sqrt{5}+1}{2}\right)}$

(d) $\frac{\sqrt{3}}{2}, \frac{1}{2}$

One or More Than One Option(s) Correct Type

7. If α, β are the roots of $ax^2 + bx + c = 0$ and $\alpha + h$, $\beta + h$ are the roots of $px^2 + qx + r = 0$, then

(a) $h = \frac{1}{2}\left(\frac{b}{a} - \frac{q}{p}\right)$ (b) $\frac{b^2 - 4ac}{a^2} = \frac{q^2 - 4pr}{p^2}$

(c) $\frac{a}{p} = \frac{b}{q} = \frac{c}{r}$ (d) none of these

8. A light ray coming along the line $3x + 4y = 5$ gets reflected from the line $ax + by = 1$ and goes along the line $5x - 12y = 10$. Then,

(a) $a = 14/15$, $b = 112/15$
(b) $a = 14/15$, $b = -8/115$
(c) $a = 64/115$, $b = -8/115$
(d) $a = 64/15$, $b = 14/15$

9. Two circles each of radius 5 units touch each other at (1, 2). If equation of their common tangent is $4x + 3y = 10$, then the equations of the circles are

(a) $x^2 + y^2 + 4x + 4y - 25 = 0$
(b) $x^2 + y^2 + 6x + 2y - 15 = 0$
(c) $x^2 + y^2 - 10x - 10y + 25 = 0$
(d) $x^2 + y^2 + 8x + 4y + 15 = 0$

10. If p and q are two statements, then $\sim(p \wedge q) \vee \sim(q \Leftrightarrow p)$ is

(a) tautology (b) contradiction
(c) neither tautology nor contradiction
(d) either tautology or contradiction

11. Three lines $x + 2y + 3 = 0$; $x + 2y - 7 = 0$ and $2x - y - 4 = 0$ form 3 sides of two squares, then the equation of remaining sides of these squares are
(a) $2x - y + 6 = 0$ (b) $x - y + 2 = 0$
(c) $2x - y - 14 = 0$ (d) $x - y - 14 = 0$

12. If $\lim_{x\to\infty}\left(1+\frac{\lambda}{x}+\frac{\mu}{x^2}\right)^{2x} = e^2$, then
(a) $\lambda = 1, \mu = 2$ (b) $\lambda = 2, \mu = 1$
(c) $\lambda = 1$, μ = any real constant
(d) $\lambda = \mu = 1$

13. The common tangents to the circle $x^2 + y^2 = 8$ and the parabola $y^2 = 16x$ are
(a) $x - y + 4 = 0$ (b) $x + y + 4 = 0$
(c) $2x + 3y + 6 = 0$ (d) $2x - 3y - 6 = 0$

Comprehension Type

If m, n, r are positive integers and if $r < m, r < n$, then
${}^mC_r + {}^mC_{r-1} . {}^nC_1 + {}^mC_{r-2} . {}^nC_2 + ... + {}^nC_r$
= Coefficient of x^r in $(1 + x)^m (1 + x)^n$
= Coefficient of x^r in $(1 + x)^{m+n} = {}^{m+n}C_r$

14. If $S_n = {}^nC_0 . {}^nC_1 + {}^nC_1 . {}^nC_2 + ... + {}^nC_{n-1} . {}^nC_n$ and if $\frac{S_{n+1}}{S_n} = \frac{15}{4}$, then n equals
(a) 2, 4 (b) 4, 6 (c) 6, 8 (d) 8, 10

15. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$ and n is odd, then the value of $C_0^2 - C_1^2 + C_2^2 - C_3^2 + ... + (-1)^nC_n^2$ is
(a) 0 (b) ${}^{2n}C_n$
(c) $(-1)^n\,{}^{2n}C_{n-1}$ (d) ${}^{2n}C_{n-2}$

Matrix Match Type

16. Match the following.

	Column I		Column II
P.	If $x_1, x_2, ..., x_{18}$ are observations such that $\sum_{j=1}^{18}(x_j - 8) = 9$ and $\sum_{j=1}^{18}(x_j - 8)^2 = 45$, then the standard deviation of these observations is	1.	25
Q.	The number of solutions of the equation $2\cos\left(\frac{x}{2}\right) = 3^x + 3^{-x}$ is	2.	1.5
R.	If odds against solving a question by three students are 2 : 1, 5 : 2 and 5 : 3 respectively, then probability that the question is solved only by one student in simplest form is $\frac{m}{n}$, then m equals to	3.	1
S.	If $f(x) = \frac{x-4}{2\sqrt{x}}$, then $f'(1)$ is	4.	1.25

	P	Q	R	S
(a)	1	3	4	2
(b)	2	3	1	4
(c)	3	1	2	4
(d)	3	2	4	1

Integer Answer Type

17. If the area of the triangle on the complex plane formed by the points z, $z + iz$ and iz is 200, then the value of $\frac{|z|}{5}$ must be equal to (where $i = \sqrt{-1}$)

18. The foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide. Then, the value of b^2 is

19. A pair of dice is thrown thrice. The probability of throwing doublets at least once is m, then value of $\frac{216}{13}m$ is

20. If $a_i > 0$, $i = 1, 2, 3,..., 50$ and $a_1 + a_2 + a_3 + ... + a_{50} = 50$, then the minimum value of $\frac{1}{25}\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+...+\frac{1}{a_{50}}\right)$ is equal to

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Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ......
No. of questions correct ......
Marks scored in percentage ......

Check your score! If your score is

> 90%	**EXCELLENT WORK !**	You are well prepared to take the challenge of final exam.
90-75%	**GOOD WORK !**	You can score good in the final exam.
74-60%	**SATISFACTORY !**	You need to score more next time.
< 60%	**NOT SATISFACTORY!**	Revise thoroughly and strengthen your concepts.

OLYMPIAD CORNER

1. For non-negative reals x, y and z satisfying $x+y+z=1$, show that $\left(\frac{1}{x}+1\right)\left(\frac{1}{y}+1\right)\left(\frac{1}{z}+1\right)\geq 64$

2. Consider the infinite sum
$$S=\frac{a_0}{10^0}+\frac{a_1}{10^2}+\frac{a_2}{10^4}+\frac{a_3}{10^6}+...,$$
where the sequence $\{a_n\}$ is defined by $a_0 = a_1 = 1$, and the recurrence relation $a_n = 20a_{n-1} + 12a_{n-2}$ for all positive integers n $\geq$ 2. If $\sqrt{S}$ can be expressed in the form $\frac{a}{\sqrt{b}}$, where a and b are relatively prime positive integers, determine the ordered pair (a, b).

3. Let S be the sum of the elements of the set $\{1, 2, 3, ..., (2p)^n - 1\}$. Let T be the sum of the elements of this set whose representation in base $2p$ consists only of digits from 0 to $p-1$. Prove that $2^n\times\frac{T}{S}=(p-1)/(2p-1)$.

4. Let p be a prime. Find all solutions in positive integers of the equation : $\frac{2}{a}+\frac{3}{b}=\frac{5}{p}$.

SOLUTIONS

1. Expanding, we first must show
$$1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}+\frac{1}{xy}+\frac{1}{xz}+\frac{1}{yz}+\frac{1}{xyz}\geq 64.$$
By AM-GM inequality,
$$xyz\leq\left(\frac{x+y+z}{3}\right)^3=\frac{1}{27}\text{ so that }\frac{1}{xyz}\geq 27.$$
Therefore,
$$1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}+\frac{1}{xy}+\frac{1}{xz}+\frac{1}{yz}+\frac{1}{xyz}$$
$$\geq 1+\frac{3}{\sqrt[3]{xyz}}+\frac{3}{\sqrt[3]{x^2y^2z^2}}+\frac{1}{xyz}$$
$$=\left(1+\frac{1}{\sqrt[3]{xyz}}\right)\geq 4^3 = 64.$$

2. We have
$$S-\frac{20S}{10^2}-\frac{12S}{10^4}=\left(\frac{a_0}{10^0}+\frac{a_1}{10^2}+\frac{a_2}{10^4}+\frac{a_3}{10^6}+....\right)$$
$$-\left(\frac{20a_0}{10^2}+\frac{20a_1}{10^4}+\frac{20a_2}{10^6}+\frac{20a_3}{10^8}+....\right)$$
$$-\left(\frac{12a_0}{10^4}+\frac{12a_1}{10^6}+\frac{12a_2}{10^8}+\frac{12a_3}{10^{10}}+....\right)$$
$$=\frac{a_0}{10^0}+\frac{a_1}{10^2}-\frac{20a_0}{10^2}+\frac{a_2-20a_1-12a_0}{10^4}$$
$$+\frac{a_3-20a_2-12a_1}{10^6}+\frac{a_4-20a_3-12a_2}{10^8}+....$$
Since, $a_n - 20a_{n-1} - 12a_{n-2} = 0$ for all positive integers $n \geq 2$, we have
$$S-\frac{20S}{10^2}-\frac{12S}{10^4}=\frac{a_0}{10^0}+\frac{a_1}{10^2}-\frac{20a_0}{10^2},$$
and substituting in $a_0 = a_1 = 1$, we have
$$\frac{7988S}{10000}=\frac{81}{100},\text{ so }S=\frac{2025}{1997}.$$
Hence, $\sqrt{S}=\frac{45}{\sqrt{1997}}$, and so the desired ordered pair is $(a, b) = (45, 1997)$.

3. We have $S=\frac{[(2p)^n-1](2p)^n}{2}$. Let R denote the set of numbers that have at most n digits in base $2p$ and contain only the digits from 0 to $p-1$. Thus T is the sum of the elements of R. For example, when $p = 2$ and

MPP CLASS XI ANSWER KEY

1.	(b)	2.	(b)	3.	(c)	4.	(b)	5.	(c)
6.	(a)	7.	(a,b)	8.	(a,c)	9.	(b,c)	10.	(c)
11.	(a,c)	12.	(c)	13.	(a,b)	14.	(a)	15.	(a)
16.	(b)	17.	(4)	18.	(7)	19.	(7)	20.	(2)

$n = 3$, we have $R = \{0, 1, 10, 11, 100, 101, 110, 111\}$ and $T = 1110_4 = 84$. Note R will have p^n elements, because each of the n digits (including leading zeros) can be any one of p different numbers. Since each number in $\{0, 1, ..., p - 1\}$ appears the same number of times as a digit in R, there will be exactly p^{n-1} elements in R that have k as their tth digit, for $k = 0, 1, ...,$ $p - 1$ and $t = 1, 2, ..., n$. Thus,

$$T = \sum_{i=0}^{n-1} (2p)^i \cdot [p^{n-1} \times 0 + p^{n-1} \times 1 + ... + p^{n-1} \times (p-1)]$$

$$= \sum_{i=0}^{n-1} (2p)^i \cdot p^{n-1} \cdot \frac{p(p-1)}{2} = \frac{p^n(p-1)}{2} \cdot \sum_{i=0}^{n-1} (2p)^i$$

$$= \frac{p^n(p-1)}{2} \cdot \frac{(2p)^n - 1}{2}$$

Hence, $$2^n \times \frac{T}{S} = \frac{2^n \cdot \frac{p^n(p-1)}{2} \cdot \frac{(2p)^n - 1}{2p-1}}{\frac{[(2p)^n - 1](2p)^n}{2}} = \frac{p-1}{2p-1}$$

as required. This can also be solved induction.

4. We have, $p(3a + 2b) = 5ab$; hence there are three cases to consider.

First case : $p = 5$. Then we get

$3a + 2b = ab$ or $(a - 2)(b - 3) = 6 = 1.6 = 2.3$

Hence, all pairs of positive integers (a, b) are $(3, 9), (4, 6), (5, 5)$ and $(8, 4)$.

Second case : p divides a. Let $a = a_1p$.

Hence, $3a_1p = b(5a_1 - 2)$.

Then p divides either b or $5a_1 - 2$. If $b = b_1p$, then $3a_1 = b_1(5a_1 - 2)$ or $(5a_1 - 2)(5b_1 - 3) = 6$ which has only one solution $(a_1, b_1) = (1, 1)$, when $(a, b) = (p, p)$. Otherwise, $5a_1 - 2 = a_2p$. Therefore,

$$3\frac{a_2p + 2}{5} = ba_2 \text{ or } 3a_2p + 6 = 5ba_2.$$

Hence, a_2 divides 6.

If $a_2 = 1$, then $(a, b) = \left(\frac{p(p+2)}{5}, \frac{3(p+2)}{5}\right)$, but only if $p \equiv 3 \pmod 5$.

If $a_2 = 2$, then $(a, b) = \left(\frac{2p(p+1)}{5}, \frac{3(p+1)}{5}\right)$, but only if $p \equiv 4 \pmod 5$.

If $a_2 = 3$, then $(a, b) = \left(\frac{p(3p+2)}{5}, \frac{3p+2}{5}\right)$, but only if $p \equiv 1 \pmod 5$.

If $a_2 = 6$, then $(a, b) = \left(\frac{2p(3p+1)}{5}, \frac{3p+1)}{5}\right)$, but only if $p \equiv 3 \pmod 5$.

Third case : p divides b. Let $b = b_1p$.

Hence, $2b_1p = a(5b_1 - 3)$

Then p divides either a or $5b_1 - 3$. If p divides a, then we have the same case (p divides a and b) as already considered above. Again (p, p) is the (only) solution. Otherwise, $5b_1 - 3 = b_2p$. Therefore,

$$2\frac{b_2p + 3}{5} = ab_2 \text{ or } 2b_2p + 6 = 5ab_2.$$

If $b_2 = 1$, then $(a, b) = \left(\frac{2(p+3)}{5}, \frac{p(p+3)}{5}\right)$, but only if $p \equiv 2 \pmod 5$.

If $b_2 = 2$, then $(a, b) = \left(\frac{2p+3}{5}, \frac{p(2p+3)}{5}\right)$, but only if $p \equiv 1 \pmod 5$.

If $b_2 = 3$, then $(a, b) = \left(\frac{2(p+1)}{5}, \frac{3p(p+1)}{5}\right)$, but only if $p \equiv 4 \pmod 5$.

If $b_2 = 6$, then $(a, b) = \left(\frac{2p+1}{5}, \frac{3p(2p+1)}{5}\right)$, but only if $p \equiv 2 \pmod 5$.

[Note that if $p = 2$ or 3, this generates only two distinct solutions : (2, 2), (1, 6) for $p = 2$ and (3,3), (12, 2) for $p = 3$. If $p > 5$, then the three solutions are all distinct.]

……

EXAM CORNER 2018

Exam	Date
VITEEE	4th to 15th April
JEE Main	8th April (Offline), 15th & 16th April (Online)
SRMJEEE	16th to 30th April
Karnataka CET	18th & 19th April
WBJEE	22nd April
Kerala PET	23rd & 24th April
MHT CET	10th May
COMEDK (Engg.)	13th May
AMU (Engg.)	13th May (Revised)
BITSAT	16th to 31st May
JEE Advanced	20th May

MATHS MUSING

SOLUTION SET-183

1. (a) : $\int_1^a [x]\, f'(x)\, dx$

$= [f(x)]_1^2 + 2[f(x)]_2^3 + \ldots + [a][f(x)]_{[a]}^a$

$= f(2) - f(1) + 2f(3) - 2f(2) + \ldots + [a]f(a) - [a]f([a])$

$= [a]f(a) - \{f(1) + f(2) + \ldots + f([a])\}$

2. (a) : According to the given condition,

$$1-\left(\frac{3}{4}\right)^n \geq \frac{9}{10}$$

$$\Rightarrow \quad \left(\frac{3}{4}\right)^n \leq 1-\frac{9}{10} = \frac{1}{10} \Rightarrow \left(\frac{4}{3}\right)^n \geq 10$$

$$\Rightarrow \quad n(\log_{10} 4 - \log_{10} 3) \geq \log_{10} 10 = 1$$

$$\therefore \; n \geq \frac{1}{\log_{10} 4 - \log_{10} 3}$$

3. (c) : Given, $\frac{dP}{dx} = (100 - 12\sqrt{x})$

$$\Rightarrow \quad dP = (100 - 12\sqrt{x})\,dx$$

On integrating both sides, we get

$\int dP = \int (100 - 12\sqrt{x})\,dx$, $P = 100x - 8x^{3/2} + C$

When $x = 0$, then $P = 2000 \Rightarrow C = 2000$

Now, when $x = 25$, then

$P = 100 \times 25 - 8 \times (25)^{3/2} + 2000 = 3500$

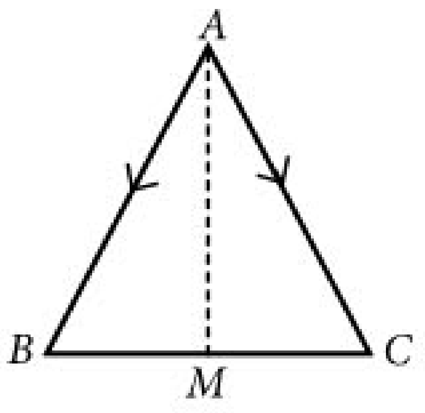
4. (b) : Given, $f(x) = \cos x - \cos^2 x + \cos^3 x - \ldots\infty$

$$= \frac{\cos x}{1 + \cos x}$$

$$\therefore \quad \int f(x)dx = \int \frac{1+\cos x}{1+\cos x}\,dx - \int \frac{1}{1+\cos x}\,dx$$

$$= x - \frac{1}{2}\int \sec^2 \frac{x}{2}\,dx = x - \frac{1}{2}\tan\frac{x}{2}\cdot 2 + C = x - \tan\frac{x}{2} + C$$

5. (c) : We know that, the sum of three vectors of a triangle is zero.



$\therefore \quad \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$

$\Rightarrow \quad \overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB}$

$$\Rightarrow \quad \overrightarrow{BM} = \frac{\overrightarrow{AC} - \overrightarrow{AB}}{2}$$

($\because$ M is mid-point of BC)

Also, $\overrightarrow{AB} + \overrightarrow{BM} + \overrightarrow{MA}\rightarrow = 0$ (by properties of a triangle)

$$\Rightarrow \quad \overrightarrow{AB} + \frac{\overrightarrow{AC} - \overrightarrow{AB}}{2} = \overrightarrow{AM}$$

$$\Rightarrow \quad \overrightarrow{AM} = \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2} = \frac{3\hat{i} + 4\hat{k} + 5\hat{i} - 2\hat{j} + 4\hat{k}}{2} = 4\hat{i} - \hat{j} + 4\hat{k}$$

$$\Rightarrow \quad |\overrightarrow{AM}| = \sqrt{4^2 + 1^2 + 4^2} = \sqrt{33}$$

6. (b, c) : $(x - \alpha)^2 + (y - \alpha)^2 = r^2$

$\Rightarrow \; x^2 + y^2 - 2\alpha x - 2\alpha y + 2\alpha^2 - r^2 = 0 \qquad$ ...(i)

Differentiating (i) w.r.t. x, we get

$2x + 2yy' - 2\alpha - 2\alpha y' = 0 \qquad$ ...(ii)

$\therefore \quad \alpha = \frac{x + yy'}{1 + y'} \qquad$ ...(iii)

Differentiating (ii) w.r.t. x, we get

$1 + y'^2 + yy'' - \alpha y'' = 0$

$\Rightarrow \; 1 + y'^2 + yy'' - \frac{x + yy'}{(1 + y')}y'' = 0 \qquad$ [From (iii)]

Collecting terms, we get

$(y - x)y'' + (1 + y' + y'^2)y' + 1 = 0$

Now compare with given differential equation.

7. (d) : $\because$ Both lines are coplanar.

$$\therefore \quad \begin{vmatrix} 2 & 3 & \lambda \\ 3 & 2 & 3 \\ 1 & -1 & -1 \end{vmatrix} = 0 \Rightarrow \lambda = 4$$

$\therefore \quad \sin^{-1} \sin 4 = \sin^{-1} \sin(\pi - 4) = \pi - 4$

8. (d) : Let $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{4} = r_1$

$\Rightarrow x = 3 + 2r_1, y = 2 + 3r_1, z = 1 + 4r_1$

It will lie on $\frac{x-2}{3} = \frac{y-3}{2} = \frac{z-2}{3}$.

$$\Rightarrow \quad \frac{3 + 2r_1 - 2}{3} = \frac{2 + 3r_1 - 3}{2} \Rightarrow r_1 = 1$$

So, point of intersection is (5, 5, 5).

9. (1) : Let $\cot^{-1}p = \theta$, then $\cot\theta = p$

$$\text{Now,} \quad e^{2mi\theta}\cdot\left(\frac{i\cot\theta + 1}{i\cot\theta - 1}\right)^m = e^{2mi\theta}\left[\frac{i(\cot\theta - i)}{i(\cot\theta + i)}\right]^m$$

$$= e^{2mi\theta}\left[\frac{\cos\theta - i\sin\theta}{\cos\theta + i\sin\theta}\right]^m = e^{2mi\theta}\left(\frac{e^{-i\theta}}{e^{i\theta}}\right)^m = e^{2mi\theta}e^{-2mi\theta} = 1$$

10. (b) : (P) → 4, (Q) → 3, (R) → 2, (S) → 1

P. $\frac{x+2}{x^2+1} > \frac{1}{2} \Rightarrow 2x + 4 > x^2 + 1$

$\Rightarrow x^2 - 2x - 3 < 0 \Rightarrow -1 < x < 3 \Rightarrow x = 0, 1, 2$

Q. If $e^x > 1 \;\Rightarrow\; 1 + e^x - 1 = e^{2x} - 2e^x$

$\Rightarrow \; e^{2x} - 3e^x = 0 \;\Rightarrow\; e^x = 3 \Rightarrow x = \log_e 3$

If $e^x < 1 \;\Rightarrow\; 1 - e^x + 1 = e^{2x} - 2e^x$

$\Rightarrow \; e^{2x} - e^x - 2 = 0 \;\Rightarrow\; (e^x - 2)(e^x + 1) = 0$

$\Rightarrow \; x = \log_e 2$, which not satisfy given equation.

R. $x = \frac{7}{2}, x = \pm 4, x = -9/2$ are four solutions.

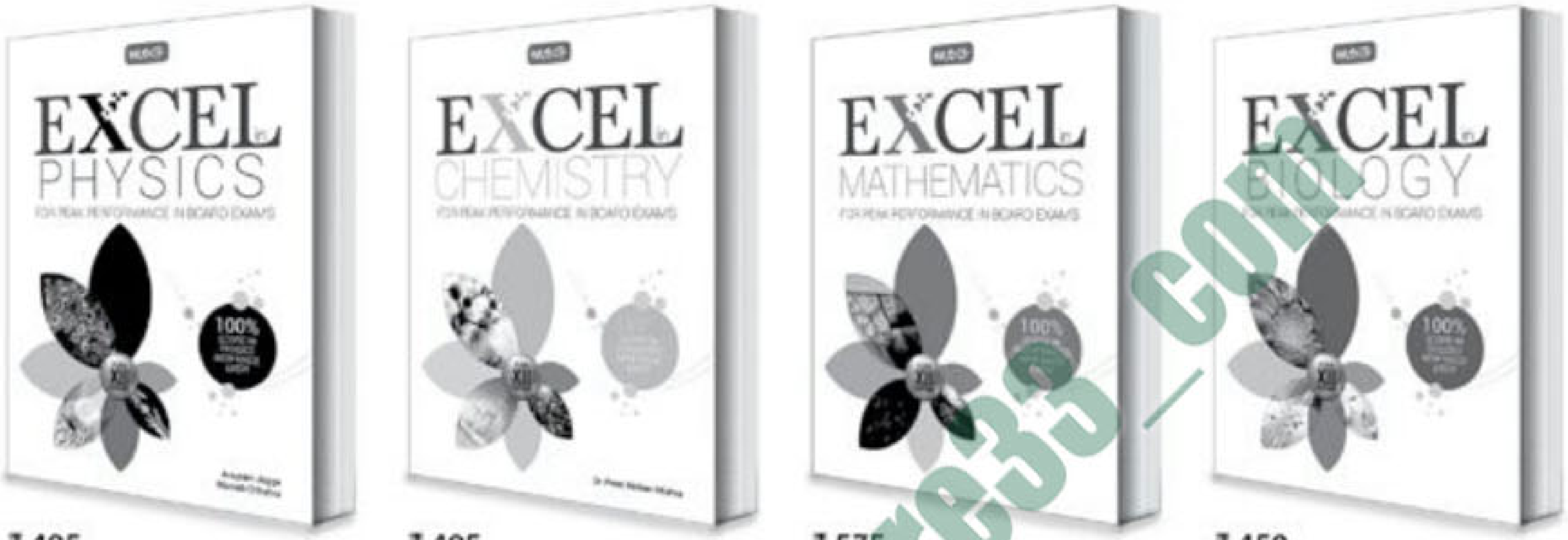
S. $-1 \leq |x - 1| - 1 \leq 1 \;\Rightarrow\; 0 \leq |x - 1| \leq 2$

$-2 \leq x - 1 \leq 2 \;\Rightarrow\; -1 \leq x \leq 3 \quad\Rightarrow\; x = -1, 0, 1, 2, 3$

There are 5 solutions.

◆◆

Concerned about your performance in Class XII Boards?



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Exam on 20th May

JEE PRACTICE PAPER 2018 ADVANCED

PAPER-I

SECTION-1

ONE OR MORE THAN ONE OPTION CORRECT TYPE

1. Which of the following set of values of 'x' satisfies the equation $2^{(2\sin^2 x-3\sin x+1)}+2^{(2-2\sin^2 x+3\sin x)}=9$

(a) $x=n\pi\pm\frac{\pi}{6}, n\in I$ (b) $x=n\pi\pm\frac{\pi}{3}, n\in I$

(c) $x = n\pi$, $n \in I$ (d) $x=2n\pi+\frac{\pi}{2}, n\in I$

2. If α, β (α < β) are 2 roots of $(6x+1)x=1+\left[\cos\frac{\pi}{4}\right]$ then $\int_0^{2\alpha}\sin\left(\frac{\pi[x]}{2}\right)dx+\int_0^{3\beta}\cos(\pi[x])\,dx=$ __ , where [.] denotes greatest integer function.

(a) α + β (b) 2

(c) 0 (d) [2α + 9β]

3. If three numbers are chosen randomly from the set {1, 3, 3^2, ..., 3^n} without replacement, then the probability that they form an increasing geometric progression is

(a) $\frac{3}{2n}$ if n is odd (b) $\frac{3}{2n}$ if n is even

(c) $\frac{3n}{2(n^2-1)}$ if n is even

(d) $\frac{3n}{2(n^2-1)}$ if n is odd

4. The general solution of the differential equation $(1+y^2)+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$ is $2xe^{f(y)}=e^{2f(y)}+c$, then the area of the region bounded by the curves $x=f(y)$, $y=\pm\sqrt{3}$ and y-axis is

(a) $\frac{\pi}{\sqrt{3}}-\log 2$ (b) $\frac{2\pi}{\sqrt{3}}-\log 4$

(c) $\frac{\pi}{\sqrt{3}}+\log 2$ (d) $\frac{2\pi}{\sqrt{3}}$

5. Given a real valued function f such that

$$f(x)=\begin{cases}\dfrac{\tan^2 x}{(x^2-[x]^2)}, & \text{for} \quad x>0\\ 1, & \text{for} \quad x=0\\ \sqrt{\{x\}\cot\{x\}}, & \text{for} \quad x<0\end{cases}$$

Where $[x]$ is the integral part and $\{x\}$ is the fractional part of x, then

(a) $\lim\limits_{x\to 0} f(x)=1$ (b) $\lim\limits_{x\to 0^-} f(x)=\sqrt{\cot 1}$

(c) $\cot^{-1}\left(\lim\limits_{x\to 0^-} f(x)\right)^2=1$

(d) f is continuous at x =0

6. A vector of magnitude 2 along a bisector of the angle between the two vectors $2\hat{i}-2\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}-2\hat{k}$ is

(a) $\frac{2}{\sqrt{10}}(3\hat{i}-\hat{k})$ (b) $\frac{1}{\sqrt{26}}(\hat{i}-4\hat{j}+3\hat{k})$

(c) $\frac{2}{\sqrt{26}}(\hat{i}-4\hat{j}+3\hat{k})$ (d) $\frac{2}{\sqrt{5}}(\hat{i}-3\hat{j})$

7. Let $f(x)=\frac{x}{1+x^2}$ and $g(x)=\frac{e^{-x}}{1+[x]}$, where [·] is the greatest integer less than or equal to x, then

(a) Domain (f + g) = R– [–2, 0)

(b) Domain (f – g) = R –[–1, 0)

(c) Range $f \cap$ Range $g=\left[-2,\frac{1}{2}\right]$

(d) Range g = R–{0}

SECTION-2

INTEGER ANSWER TYPE

8. The number of acute triangles whose vertices are chosen from the vertices of a rectangular block are

9. Tangents are drawn to the circle $x^2 + y^2 = 12$ at the points where it is meet by the circle $x^2 + y^2 - 5x + 3y - 2 = 0$, then the x-coordinate of the point of intersection of these tangents is

10. Let $a_1, a_2, ..., a_n$ be an A.P. with common difference $\pi/6$ and assume

$$\sec a_1 \sec a_2 + \sec a_2 \sec a_3 + ... + \sec a_{n-1} \sec a_n = k(\tan a_n - \tan a_1).$$

Find the value of k.

11. If $f(x) = x^2 - x + 1, x \geq \frac{1}{2}$ and $g(x) = \frac{1}{2} + \sqrt{x - \frac{3}{4}}$ are two functions, then the number of solutions of the equation $x^2 - x + 1 = \frac{1}{2} + \sqrt{\left(x - \frac{3}{4}\right)}$ is

12. If $f(x + y) = f(x) f(y)$ and $f(x) = 1 + xg(x)H(x)$ where $\lim\limits_{x \to 0} g(x) = 2$, $\lim\limits_{x \to 0} H(x) = 3$, then $f'(x) = Kf(x)$ if $K =$

SECTION-3

MATRIX MATCH TYPE

13. Match the following.

	Column-I		Column-II
(A)	Radius of the circle passing through (3, 4) and (5, 2) having centre on $y = 2x$ is	(p)	4
(B)	Length of the common chord of $2y^2 = 3(x + 1)$, $x^2 + y^2 + 2x = 0$ is	(q)	$\sqrt{52}$
(C)	Product of length of perpendiculars drawn from the two foci to the tangent at any point on the ellipse $25x^2 + 4y^2 = 100$ is	(r)	$\sqrt{3}$

14. Match the following.

	Column-I		Column-II
(A)	If $I_1 = \int_0^1 x^4(1-x)^5 dx$, $I_2 = \int_0^1 x^5(1-x)^5 dx$ then $\frac{I_2}{I_1} =$	(p)	$\frac{1}{2}$
(B)	If $I_1 = \int_0^{\pi} \cos^4 x \sin^6 x dx$, $I_2 = \int_0^{2\pi} \cos^4 x \sin^4 x dx$ then $\frac{I_1}{I_2} =$	(q)	$\frac{1}{3}$
(C)	If $f(x) = \frac{\sin^2 x}{(1-e^{-3x})\sin^2 2x}$ then $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} f(x) dx =$	(r)	$\frac{1}{4}$
(D)	$\int_0^{\frac{\pi}{2}} (\sqrt{\sin x} + \sqrt{\cos x})^{-4} dx =$	(s)	$\frac{5}{11}$

15. Match the following:

	Column-I		Column-II
(A)	The no. of solutions of the equation $|\cos x| = 2[x]$, (where $[\cdot]$ is greatest integer function) is	(p)	8
(B)	The no. of solutions of the equation $2^{\cos x} = |\sin x|$ in $[-2\pi, 2\pi]$ is	(q)	4
(C)	The no. of solutions of the equation $\sin^3 x \cos x + \sin^2 x \cos^2 x + \sin x \cos^3 x = 1$ in $[0, 2\pi]$ are	(r)	0
(D)	The number of possible values of k if fundamental period of $\sin^{-1}(\sin kx)$ is $\pi/2$ is	(s)	2

Solution Sender of Maths Musing

SET-183

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N. Jayanthi	(Hyderabad)

PAPER-II

SECTION-1

SINGLE OPTION CORRECT TYPE

1. Through the point $P(h, k, l)$ a plane is drawn at right angles to OP to meet co-ordinate axes at A, B and C. If $OP = p$, then the area of the ΔABC is

(a) $\frac{p^5}{2hkl}$ (b) $\frac{p^5}{hkl}$ (c) $\frac{p^3}{2hkl}$ (d) $\frac{p^3}{hkl}$

2. Sixteen players $s_1, s_2, \ldots, s_{16}$ play in a tournament. They are divided into eight pairs at random. From each pair a winner is decided on the basis of a game played between the two players of the pair. Assume that all the players are of equal strength. The probability that "exactly one of the two players s_1 and s_2 is among the eight winners" is

(a) 4/15 (b) 7/15 (c) 8/15 (d) 9/15

3. The solution of the differential equation

$$\frac{x\,dx + y\,dy}{x\,dy - y\,dx} = \sqrt{\frac{a^2 - x^2 - y^2}{x^2 + y^2}} \text{ is}$$

(a) $\sqrt{x^2 + y^2} = a\cos\left\{c + \tan^{-1}\frac{y}{x}\right\}$

(b) $\sqrt{x^2 + y^2} = a\sin\left\{c + \tan^{-1}\frac{y}{x}\right\}$

(c) $\sqrt{x^2 + y^2} = a\sin\left\{c + \tan^{-1}\frac{x}{y}\right\}$

(d) $\sqrt{x^2 + y^2} = a\cos\left\{c + \tan^{-1}\frac{x}{y}\right\}$

4. In a certain test there are n questions. In this test 2^{n-i} students gave wrong answers to atleast i questions, where $i = 1, 2, 3, \ldots, n$. If the total number of wrong answers given is 2047, then n is

(a) 10 (b) 11 (c) 12 (d) 13

5. Let three matrices $A = \begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix}; B = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$, then $tr(A) + tr\left(\frac{ABC}{2}\right)$

$$+\, tr\left(\frac{A(BC)^2}{4}\right) + tr\left(\frac{A(BC)^3}{8}\right) + \ldots\ldots + \infty =$$

(a) 6 (b) 9

(c) 12 (d) none of these

6. If a, b, c and d are distinct positive real numbers such that a and b are the roots of $x^2 - 10cx - 11d = 0$ and c and d are the roots of $x^2 - 10ax - 11b = 0$, then the value of $a + b + c + d$ is

(a) 1110 (b) 1010 (c) 1101 (d) 1210

7. The square $ABCD$ where $A(0, 0)$, $B(2, 0)$, $C(2, 2)$, $D(0, 2)$ undergoes the following transformations successively

(i) $f_1(x, y) = (y, x)$ (ii) $f_2(x, y) = (x + 3y, y)$

(iii) $f_3(x, y) = \left(\frac{x - y}{2}, \frac{x + y}{2}\right)$. Then the final figure is

(a) a square (b) a rectangle

(c) a parallelogram (d) a rhombus

SECTION-2

ONE OR MORE THAN ONE OPTION CORRECT TYPE

8. If $\cos x + \cos y = a, \cos 2x + \cos 2y = b, \cos 3x + \cos 3y = c$, then

(a) $\cos^2 x + \cos^2 y = 1 + \frac{b}{2}$

(b) $\cos x \cos y = \frac{a^2}{2} - \left(\frac{b+2}{4}\right)$

(c) $2a^3 + c = 3a(1 + b)$

(d) $a + b + c = 3abc$

9. If $f(x) = \int_x^{x^2} \frac{dt}{(\log t)^2}, x \neq 0,\ x \neq 1,$ then $f(x)$ is

(a) monotonically increasing in $(2, \infty)$

(b) monotonically increasing in $(1, 2)$

(c) monotonically decreasing in $(2, \infty)$

(d) monotonically decreasing in $(0, 1)$

10. Let $A = \int_{e^{-1}}^{\tan x} \frac{t\,dt}{t^2 + 1}$ and $B = \int_{e^{-1}}^{\cot x} \frac{dt}{t(t^2 + 1)}$, then

(a) At $x = \frac{\pi}{4}, A + B = 1$

(b) $A + B = 1$ for all x in $\left(0, \frac{\pi}{2}\right)$

(c) $A + B = 1$ for all x in $\left(0, \frac{\pi}{4}\right)$ and 2 for all x in $\left(\frac{\pi}{4}, \pi\right)$

(d) $A = B$ for all x

11. If $h(x)=3f\left(\frac{x^2}{3}\right)+f(3-x^2), \forall x \in (3,4)$, where $f''(x)>0,\ \forall\ x\in(-3,\ 4)$, then $h(x)$ is
(a) increasing in (3/2, 4)
(b) increasing in (–3/2, 0)
(c) decreasing in (0, 3/2)
(d) None of these

12. The eccentric angles of extremities of a chord of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are θ_1 and θ_2. If this chord passes through the focus, then
(a) $\tan\frac{\theta_1}{2}\cdot\tan\frac{\theta_2}{2}+\frac{1-e}{1+e}=0$
(b) $\cos\left(\frac{\theta_1-\theta_2}{2}\right)=e\cos\left(\frac{\theta_1+\theta_2}{2}\right)$
(c) $\cot\frac{\theta_1}{2}\cdot\cot\frac{\theta_2}{2}=\frac{e+1}{e-1}$
(d) None of these

13. The normal at a general point $(a,\ b)$ on curve makes an angle θ with x-axis which satisfies $b(-a^2\tan\theta-\cot\theta)=a(b^2+1)$. The equation of curve can be
(a) $y = e^{x^2/2} + c$ (b) $\log(ky^2) = x^2$
(c) $y = ke^{x^2/2}$ (d) $x^2 - y^2 = k$

14. If $S_r=\begin{vmatrix} 2r & x & n(n+1) \\ 6r^2-1 & y & n^2(2n+3) \\ 4r^3-2nr & z & n^3(n+1)\end{vmatrix}$, then the value $\sum_{r=1}^{n} S_r$ is independent of
(a) x (b) y (c) n (d) z

SECTION-3

COMPREHENSION TYPE

Paragraph for Question No. 15 and 16

Let A, B, C be three sets of complex numbers as defined below,

$A=\{z : \text{Im } z \geq 1\}$; $B=\{z : |z-2-i|=3\}$

$C=\{z : \text{Re}((1-i)z)=\sqrt{2}\}$

15. The number of elements in the set $A \cap B \cap C$ is
(a) 0 (b) 1 (c) 2 (d) ∞

16. Let z be any point in $A \cap B \cap C$ and let w be any point satisfying $|w-2-i|<3$. Then, $|z|-|w|+3$ lies between
(a) –6 and 3 (b) –3 and 6
(c) –6 and 6 (d) –3 and 9

Paragraph for Question No. 17 and 18

Define a function $\phi : N \to N$ as follows : $\phi(1)=1$, $\phi(p^n)=p^{n-1}$, if p is prime and $n \in N$. $\phi(mn)=\phi(m)\,\phi(n)$ if m and n are relatively prime natural numbers.

17. $\phi(8n+4)$ where $n \in N$ is equal to
(a) $\phi(4n+2)$ (b) $\phi(2n+1)$
(c) $2\phi(2n+1)$ (d) $4\phi(2n+1)$

18. The number of natural numbers 'n' such that $\phi(n)$ is odd is
(a) 1 (b) 2 (c) 3 (d) 4

SOLUTIONS

PAPER-I

1. (a, d) : $2^{(2\sin^2 x-3\sin x+1)}+2^{3-(2\sin^2 x-3\sin x+1)}=9$

Let $2^{(2\sin^2 x-3\sin x+1)}=t$

$\Rightarrow t+\frac{8}{t}=9 \Rightarrow t^2-9t+8=0 \Rightarrow t=1,\ 8$

$\Rightarrow\ 2\sin^2 x-3\sin x+1=3$ or $2\sin^2 x-3\sin x+1=0$

$\Rightarrow \sin x=-\frac{1}{2}, \sin x=\frac{1}{2}, \sin x=1$

2. (b,d) : $(6x+1)x=1+\left[\cos\frac{\pi}{4}\right]$

$\Rightarrow\ 6x^2+x-1=0\ \Rightarrow\ x=\frac{-1}{2},\ \frac{1}{3};\ \alpha=-\frac{1}{2},\ \beta=\frac{1}{3}$

$\therefore\ I=\int_0^{-1}\sin\frac{\pi[x]}{2}dx+\int_0^1\cos(\pi[x])dx=\int_{-1}^{0}1dx+\int_0^1 1dx=2$

Also $[2\alpha+9\beta]=[-1+3]=2$

3. (a,c) : Number of triplets

$(3^r, 3^{r+1}, 3^{r+2})\ (0 \leq r \leq n-2)$ is $n-1$

Number of triplets

$(3^r, 3^{r+2}, 3^{r+4})(0 \leq r \leq n-4)$ is $n-3$

. . .
. . .
. . .
. . .

Number of triplets, $\left(3^r, 3^{r+\frac{n-1}{2}}, 3^{r+n-1}\right)(n \text{ odd})$ is 2

Number of triplets, $\left(3^r, 3^{r+\frac{n}{2}}, 3^{r+n}\right)(n \text{ even})$ is 1

$\therefore$ If n is odd, the number of favourable outcomes

$=(n-1)+(n-3)+....+4+2=\frac{n^2-1}{4}$

and if n is even, the number of favourable outcomes

$=(n-1)+(n-3)+....+3+1=\frac{n}{2}\times\frac{n}{2}=\frac{n^2}{4}$

$\therefore$ Required Probability $= \dfrac{(n^2-1)/4}{^{(n+1)}C_3} = \dfrac{3}{2n}$, if n is odd

or $\dfrac{n^2/4}{^{(n+1)}C_3} = \dfrac{3n}{2(n^2-1)}$ if n is even

4. (b) : $\dfrac{dx}{dy} + \dfrac{x}{1+y^2} = \dfrac{e^{\tan^{-1}y}}{1+y^2}$

$$\text{I.F.} = e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$$

$\therefore$ General solution is $x \cdot e^{\tan^{-1}y} = \displaystyle\int \frac{\left(e^{\tan^{-1}y}\right)^2}{1+y^2}dy$

$$\Rightarrow \quad x.e^{\tan^{-1}y} = \frac{e^{2\tan^{-1}y}}{2} + c_1$$

$$\Rightarrow \quad 2xe^{\tan^{-1}y} = e^{2\tan^{-1}y} + c \quad \therefore \quad f(y) = \tan^{-1}y$$

$$\therefore \quad \text{Area} = \int_{-\sqrt{3}}^{\sqrt{3}} |\tan^{-1}y|\,dy = 2\int_0^{\sqrt{3}} \tan^{-1}y\,dy$$

$$= 2\left[(y\tan^{-1}y)_0^{\sqrt{3}} - \int_0^{\sqrt{3}} \frac{y}{1+y^2}dy\right]$$

$$= 2\frac{\pi}{\sqrt{3}} - \left[\log(1+y^2)\right]_0^{\sqrt{3}} = \frac{2\pi}{\sqrt{3}} - \log 4$$

5. (b, c) :

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\tan^2 x}{(x^2-[x]^2)} = \lim_{x\to 0^+} \frac{\tan^2 x}{x^2} = 1$$

As $x \to 0^+$, $[x] = 0$

Also, $\displaystyle\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \sqrt{(x-[x])\cot(x-[x])} = \sqrt{\cot 1}$

$$\cot^{-1}\left(\lim_{x\to 0^-} f(x)\right)^2 = \cot^{-1}(\cot 1) = 1$$

6. (a, c) : Let $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

$\therefore \quad |\vec{a}| = 3, \left|\vec{b}\right| = 3$

$\therefore$ Vector along bisectors is given by

$$= \frac{\vec{a}}{|\vec{a}|} \pm \frac{\vec{b}}{\left|\vec{b}\right|} = \frac{2\hat{i} - 2\hat{j} + \hat{k}}{3} \pm \frac{\hat{i} + 2\hat{j} - 2\hat{k}}{3}$$

$$= \hat{i} - \frac{1}{3}\hat{k}, \frac{1}{3}\hat{i} - \frac{4}{3}\hat{j} + \hat{k}$$

$\therefore$ Required vectors are

$$= 2.\frac{\hat{i} - \frac{1}{3}\hat{k}}{\sqrt{1^2 + \left(-\frac{1}{3}\right)^2}}, 2.\frac{\frac{1}{3}\hat{i} - \frac{4}{3}\hat{j} + \hat{k}}{\sqrt{\left(\frac{1}{3}\right)^2 + \left(-\frac{4}{3}\right)^2 + 1^2}}$$

7. (b, d) : Domain of $f = R$, Domain $g = R - [-1, 0)$; because $-1 \le x < 0$ thus $1 + [x] = 0$

$\therefore$ Domain of $f - g = R - [-1, 0)$

since $e^{-x} > 0 \Rightarrow (1 + [x])y > 0$

Either $y > 0 \Rightarrow 1 + [x] > 0$, or $y < 0 \Rightarrow 1 + [x] < 0$

$\therefore \quad y \in R - \{0\}$

8. (8) : The 8 vertices of the block gives $^8C_3 = 56$ triangles, each of which is either acute or right angled. Each vertex of the block serves as the vertex of the right angle in three triangles whose hypotenuse are face diagonals of the block, and three triangles whose hypotenuse are space diagonals of the block. Hence there are $8(3 + 3) = 48$ right triangles and so $56 - 48 = 8$ acute triangles.

9. (6) : The circles are given as $x^2 + y^2 = 12$ and $x^2 + y^2 - 5x + 3y - 2 = 0$

Common chord AB is $5x - 3y - 10 = 0$. Let the coordinates of P be (α, β)

Equation of the chord of contact of $F(\alpha, \beta)$ with respect to $x^2 + y^2 = 12$ is $x\alpha + y\beta - 12 = 0$ comparing the coefficients of common chord AB and chord of contact is $\dfrac{\alpha}{5} = \dfrac{\beta}{-3} = \dfrac{-12}{-10} \Rightarrow \alpha = 6$

$\therefore$ x-coordinate is 6.

10. (2) :

$$\frac{1}{\cos a_1 \cos a_2} + \frac{1}{\cos a_2 \cos a_3} + \ldots\ldots + \frac{1}{\cos a_{n-1} \cos a_n}$$

$$\Rightarrow \frac{1}{\sin d}\left(\frac{\sin(a_2-a_1)}{\cos a_1 \cos a_2} + \frac{\sin(a_3-a_2)}{\cos a_2 \cos a_3} + \ldots + \frac{\sin(a_n-a_{n-1})}{\cos a_{n-1}\cos a_n}\right)$$

$$= \frac{1}{\sin d}((\tan a_2 - \tan a_1) + (\tan a_3 - \tan a_2) + \ldots + (\tan a_n - \tan a_{n-1}))$$

$$= \frac{1}{\sin d}(\tan a_n - \tan a_1) \Rightarrow k = \frac{1}{\sin d} = 2$$

11. (1) : The function $y = f(x) = x^2 - x + 1$

$= \left(x - \dfrac{1}{2}\right)^2 + \dfrac{3}{4}$ increases in the interval $\left[\dfrac{1}{2}, \infty\right)$ and x varying in the indicated interval we have

$$y = f(x) \ge \frac{3}{4} \text{ i.e., } y \in \left[\frac{3}{4}, \infty\right)$$

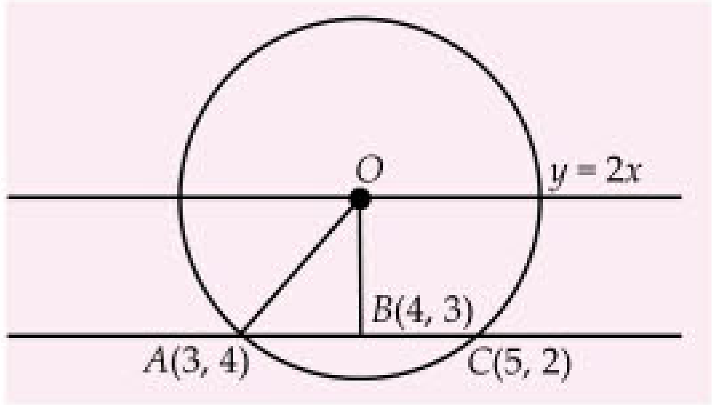
12. (6) : $f'(x) = \displaystyle\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h\to 0} \frac{f(x)f(h) - f(x)}{h} = \lim_{h\to 0} \frac{f(x)[1 + h\,g(h)H(h) - 1]}{h}$$

$$= f(x)\lim_{h\to 0} g(h) \cdot H(h) = 6f(x)$$

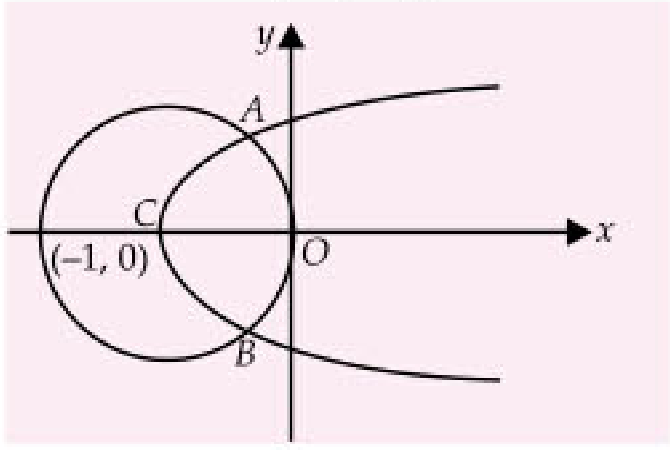
13. A – q; B – r; C – p

(A) Equation of perpendicular bisector of side AC is $x - y = 1$



So, centre O is $(-1,-2)$; Radius $= \sqrt{52}$

(B)



Point of intersection of the two curves are

$$A\left(\frac{-1}{2},\frac{\sqrt{3}}{2}\right) \text{ and } B\left(\frac{-1}{2},-\frac{\sqrt{3}}{2}\right)$$

So, common chord length $AB = \sqrt{3}$.

(C) $P_1P_2 = b^2$, $\frac{x^2}{4}+\frac{y^2}{25}=1$, so $P_1P_2 = 4$

14. A → s; B → r; C → r; D → q

(A) $I_1 = \frac{4!.5!}{10!}$, $I_2 = \frac{5!.5!}{11!} \Rightarrow \frac{I_2}{I_1} = \frac{5}{11}$

(B) $I_1 = \frac{3.5.3}{10.8.6.4.2}.\pi$, $I_2 = \frac{3.3}{8.6.4.2}2\pi \Rightarrow \frac{I_1}{I_2} = \frac{1}{4}$

(C) $\int_{-a}^{a} f(x)dx = \int_0^a (f(x)+f(-x))dx$

$$\therefore \quad I = \int_0^{\frac{\pi}{4}} \frac{\sin^2 x}{\sin^2 2x}\left(\frac{1}{1-e^{-3x}}+\frac{1}{1-e^{3x}}\right)dx$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{4}} \sec^2 x dx = \frac{1}{4}$$

(D) $\tan x = t \Rightarrow I = \int_0^{\infty} \frac{dt}{(\sqrt{t}+1)^4}$

$\sqrt{t} = u - 1$

$$\Rightarrow \quad I = 2\int_1^{\infty} \frac{u-1}{u^4}du = 2\int_1^{\infty}(u^{-3}-u^{-4})du = \frac{1}{3}$$

15. A - r; B - p; C - r; D - q

(A) $y = |\cos x|$, $y = 2[x]$

$\therefore$ Number of solutions will be 0.

(B) The number of solutions is 8

(C) $\sin^3 x\cos x + \sin^2 x\cos^2 x + \sin x\cos^3 x = 1$

$\Rightarrow \sin x\cos x(\sin^2 x + \sin x\cos x + \cos^2 x) = 1$

$$\Rightarrow \frac{\sin 2x}{2}\left(1+\frac{\sin 2x}{2}\right)=1$$

$\Rightarrow \sin 2x(2 + \sin 2x) = 4 \Rightarrow \sin^2 2x + 2\sin 2x - 4 = 0$

$$\Rightarrow \sin 2x = \frac{-2 \pm \sqrt{4+16}}{2} = -1 \pm \sqrt{5} \text{ (Impossible)}$$

(D) $\frac{2\pi}{|k|} = \frac{\pi}{2} \Rightarrow |k| = 4$

PAPER-II

1. (a) : Equation of the plane through $P(h, k, l)$ perpendicular to OP is

$xh + yk + zl = h^2 + k^2 + l^2 = p^2$; where, $p^2 = h^2 + k^2 + l^2$

$$\Rightarrow \frac{x}{\frac{p^2}{h}}+\frac{y}{\frac{p^2}{k}}+\frac{z}{\frac{p^2}{l}}=1$$

$$\Delta_{xy} = \frac{1}{2}\cdot\frac{p^2}{h}\cdot\frac{p^2}{k}, \Delta_{yz} = \frac{1}{2}\cdot\frac{p^2}{k}\cdot\frac{p^2}{l}, \Delta_{zx} = \frac{1}{2}\cdot\frac{p^2}{l}\cdot\frac{p^2}{h}$$

$$A = \sqrt{(\Delta_{xy})^2+(\Delta_{yz})^2+(\Delta_{zx})^2} = \frac{p^4}{2}\sqrt{\frac{l^2+h^2+k^2}{h^2k^2l^2}}$$

$$= \frac{p^4}{2}\sqrt{\frac{p^2}{h^2k^2l^2}} = \frac{p^5}{2hkl}$$

Hence, $Ar\,(\Delta ABC) = \frac{p^5}{2hkl}$

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2. (c) : Let

E_1 : s_1 and s_2 are in the same group

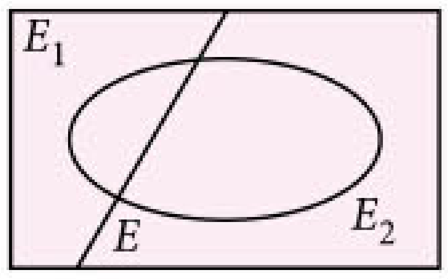
E_2 : s_1 and s_2 are in the different group

E : exactly one of the two players s_1 and s_2 is among the eight winners.

$$E = (E \cap E_1) \cup (E \cap E_2)$$

$$\Rightarrow P(E) = P(E \cap E_1) + P(E \cap E_2)$$

$$\Rightarrow P(E) = P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2)$$



Now $P(E_1) = \dfrac{\dfrac{(14)!}{(2)^7 \cdot 7!}}{\dfrac{16!}{2^8 \cdot 8!}} = \dfrac{1}{15}$

$$P(E_2) = 1 - \frac{1}{15} = \frac{14}{15}$$

Hence $P(E) = \dfrac{1}{15} \cdot 1 + \dfrac{14}{15} \cdot P$ (exactly one of either S_1 or S_2 wins)

$$= \frac{1}{15} + \frac{14}{15} \cdot \left(\frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \right) = \frac{1}{15} + \frac{14}{15} \cdot \frac{1}{2} = \frac{1}{15} + \frac{7}{15} = \frac{8}{15}.$$

3. (b): Put $x = r\cos\theta$, $y = r\sin\theta$

$$\Rightarrow x^2 + y^2 = r^2, \tan\theta = \frac{y}{x}$$

$$\Rightarrow xdx + ydy = rdr \text{ and } \frac{x\,dy - y\,dx}{x^2} = \sec^2\theta\, d\theta$$

$$\therefore \text{ Given equation } \Rightarrow \frac{r\,dr}{r^2 d\theta} = \sqrt{\frac{a^2 - r^2}{r^2}}$$

$$\Rightarrow \frac{dr}{\sqrt{a^2 - r^2}} = d\theta \Rightarrow \sin^{-1}\left(\frac{r}{a}\right) = \theta + c$$

$$\Rightarrow r = a\sin(\theta + c)$$

4. (b): Number of students who gave wrong answers to atleast i questions $= 2^{n-i}$

Number of students who gave wrong answers to atleast n questions $= 2^0 = 1$

Number of students gave wrong answers to exactly i questions $= 2^{n-i} - 2^{n-(i+1)}$

Number of wrong answers

$$= \sum_{i=1}^{n-1} i\left(2^{n-i} - 2^{n-(i+1)}\right) + (n) = 2047$$

$$\Rightarrow 1 + 2^1 + 2^2 + \ldots + 2^{n-1} = 2047$$

$$\Rightarrow 2^n = 2048 \Rightarrow n = 11$$

5. (a) : $BC = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix} \Rightarrow BC = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$$\therefore tr(A) + tr\left(\frac{A}{2}\right) + tr\left(\frac{A}{2^2}\right) + \ldots\ldots$$

$$= tr(A) + \frac{1}{2} tr(A) + \frac{1}{2^2} tr(A) + \ldots$$

$$= \frac{tr(A)}{1 - (1/2)} = 2tr(A) = 2(2+1) = 6$$

6. (d): Since a and b are the roots of $x^2 - 10cx - 11d = 0$, we have

(i) $a + b = 10c$ and (ii) $ab = -11d$ ... (1)

Also, since c and d are the roots of $x^2 - 10ax - 11b = 0$, we have

(i) $c + d = 10a$ and (ii) $cd = -11b$ ... (2)

Adding part (i) of Eqs. (1) and (2), we get

$a + b + c + d = 10(a + c)$

$\Rightarrow b + d = 9(a + c)$ ... (3)

Multiplying part (ii) of Eqs. (1) and (2), we get

$abcd = 121bd \Rightarrow ac = 121$ ... (4)

Also, $a^2 - 10ca - 11d = 0 = c^2 - 10ca - 11b$

$\Rightarrow a^2 + c^2 - 20ca - 11(b + d) = 0$

From Eqs. (3) and (4), we have

$a^2 + c^2 - 20(121) - 99(a + c) = 0$

$\Rightarrow (a + c)^2 - 2 \times 121 - 20 \times 121 - 99(a + c) = 0$

$\Rightarrow (a + c - 121)(a + c + 22) = 0$

$\Rightarrow a + c = 121$; $a + c = -22$

Since a, c are positive, $a + c \neq -22$. Therefore $a + c = 121$ and

$a + b + c + d = (a + c) + 9(a + c) = 1210$

7. (c) : $f_3 \cdot f_2 \cdot f_1(x, y) = f_3 \cdot f_2(y, x) = f_3(y + 3x, x)$

$$= \left(\frac{y + 3x - x}{2}, \frac{y + 3x + x}{2} \right) = \left(\frac{2x + y}{2}, \frac{4x + y}{2} \right)$$

$\therefore$ $A\ (0, 0) \rightarrow A'|(0,0)$, $B\ (2, 0) \rightarrow B'|\ (2, 4)$, $C\ (2, 2) \rightarrow C'(3, 5)$, $D(0, 2) \rightarrow D'(1, 1)$

$\therefore$ It is easily seen $A'B' = D'C'$, $A'D' = B'C'$, $A'B' \neq B'C'$.

Hence, it is neither a square nor a rhombus.

Further slope of $A'B' \times A'D' = \dfrac{4}{2} \cdot \dfrac{1}{1} \neq -1$

Hence AB is not perpendicular to AD.

$\Rightarrow$ It is not a rectangle

$\therefore$ $ABCD$ is a parallelogram.

8. (a, b, c) : $(\cos x + \cos y)^2 = a^2$

$\Rightarrow \cos^2 x + \cos^2 y + 2\cos x \cos y = a^2$ ... (1)

$\cos 2x + \cos 2y = b$

$\Rightarrow 2\cos^2 x - 1 + 2\cos^2 y - 1 = b$

$\Rightarrow\ 2[\cos^2 x + \cos^2 y] = b + 2$

$\Rightarrow\ \cos^2 x + \cos^2 y = \dfrac{b}{2}+1$ ... (2)

From (1) and (2),

$$2\cos x\cos y = a^2 - \left(\frac{b+2}{2}\right)$$

$$\Rightarrow\ \cos x\cos y = \frac{a^2}{2} - \left(\frac{b+2}{4}\right)$$

$\cos 3x + \cos 3y = c$

$\Rightarrow\ 4\cos^3 x - 3\cos x + 4\cos^3 y - 3\cos y = c$

$\Rightarrow\ 4[\cos^3 x + \cos^3 y] - 3[\cos x + \cos y] = c$

$\Rightarrow\ 4[(\cos x + \cos y)(\cos^2 x + \cos^2 y - \cos x\cos y)]$
$- 3(\cos x + \cos y) = c$

$$\Rightarrow\ 4\left[a\left(\frac{b+2}{2} - \frac{1}{2}\left(a^2 - \frac{b+2}{2}\right)\right)\right] - 3a = c$$

$\Rightarrow\ 2ab + 4a - 2a^3 + ab + 2a = 3a + c$

$\Rightarrow\ 2a^3 + c = 3a(1 + b)$

9. (a,d): $f'(x) = \dfrac{2x}{(\log x^2)^2} - \dfrac{1}{(\log x)^2} = \dfrac{1}{(\log x)^2}\left[\dfrac{x}{2} - 1\right]$

$\Rightarrow\ f'(x) > 0 \Rightarrow x \in (2, \infty)$

$\Rightarrow\ f'(x) < 0 \Rightarrow x \in (0, 2)$

10. (a, b) : Let $A + B = f(x)$

$$= \int_{e^{-1}}^{\tan x} \frac{t\,dt}{t^2+1} + \int_{e^{-1}}^{\cot x} \frac{dt}{t(1+t^2)}$$

$$\Rightarrow f'(x) = \frac{\tan x}{\tan^2 x + 1}\cdot\sec^2 x + \frac{1}{\cot x(1+\cot^2 x)}(-\mathrm{cosec}^2 x) = 0$$

$$\Rightarrow f(x) \text{ is constant } \Rightarrow f(x) = f\left(\frac{\pi}{4}\right)$$

$$\Rightarrow f(x) = \int_{e^{-1}}^{1} \frac{t\,dt}{t^2+1} + \int_{e^{-1}}^{1} \frac{dt}{t(t^2+1)} = \int_{e^{-1}}^{1} \frac{dt}{t} = 1$$

$$\Rightarrow f(x) = 1 \text{ for all } x \text{ in} \left(0, \frac{\pi}{2}\right)$$

11. (a, b, c) : $h'(x) = 2x\{f'(x^2/3) - f'(3-x^2)\}$

$f'(x^2/3) > f'(3-x^2)\ \forall x$ such that

$$\frac{x^2}{3} > 3 - x^2\ \Rightarrow\ x^2 > \frac{9}{4}$$

$f'(x^2/3) < f'(3-x^2)\ \forall x$ such that $x^2 < 9/4$

$h(x)$ increases in $(-3/2, 0) \cup (3/2, 4)$ and $h(x)$ decreases in $(0, 3/2)$

12. (a,b,c) : Equation of chord PQ is

$$\frac{x}{a}\cos\left(\frac{\theta_1+\theta_2}{2}\right) + \frac{y}{b}\sin\left(\frac{\theta_1+\theta_2}{2}\right) = \cos\left(\frac{\theta_1-\theta_2}{2}\right)$$

This passes through focus $(ae, 0)$

$$\therefore\ e = \frac{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}{\cos\left(\frac{\theta_1+\theta_2}{2}\right)}\ \Rightarrow\ \frac{e+1}{e-1} = \cot\frac{\theta_1}{2}\cot\frac{\theta_2}{2}$$

$$\Rightarrow\ \tan\left(\frac{\theta_1}{2}\right)\tan\left(\frac{\theta_2}{2}\right) + \frac{1-e}{e+1} = 0$$

13. (b, c, d) : Slope of normal, $\tan\theta = -\dfrac{dx}{dy}$

$\therefore$ The given equation becomes at a general point (x, y)

$$y\left(-x^2\left(-\frac{dx}{dy}\right) + \frac{dy}{dx}\right) = x(y^2+1)$$

$$\Rightarrow\ yx^2 + y\left(\frac{dy}{dx}\right)^2 = \frac{dy}{dx}.x(y^2+1)$$

$$\Rightarrow\ y\left(\frac{dy}{dx}\right)^2 - x(y^2+1)\frac{dy}{dx} + yx^2 = 0$$

$\Rightarrow\ yy'^2 - xy^2y' - xy' + yx^2 = 0$

$\Rightarrow\ yy'(y' - xy) - x(y' - xy) = 0$

$$\therefore\ \frac{dy}{dx} = xy \text{ or } \frac{dy}{dx} = \frac{x}{y}$$

$$\Rightarrow\ \log y = \frac{x^2}{2} + c \text{ or } x^2 - y^2 = c$$

$$\Rightarrow\ y = ke^{\frac{x^2}{2}} \text{ or } \log y^2 = x^2 - \log k$$

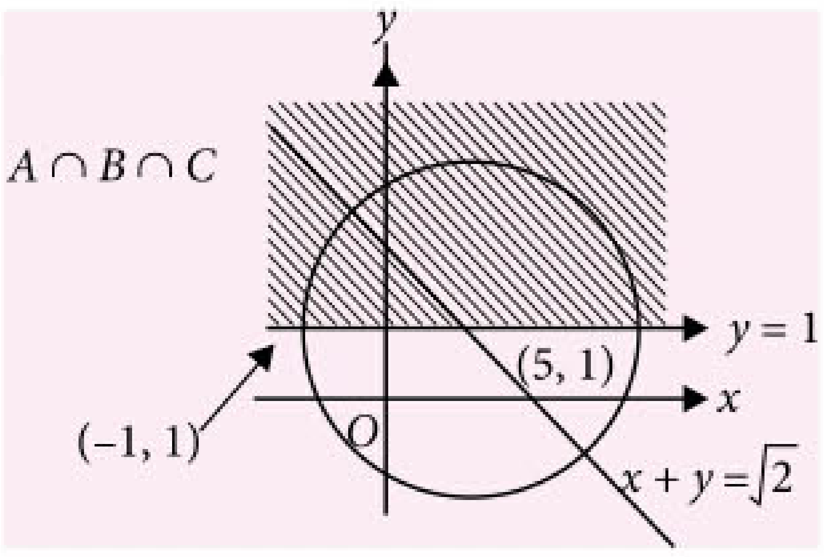
$\Rightarrow\ \log(ky^2) = x^2$

14. (a, b, c, d) : $\displaystyle\sum_{r=1}^{n} S_r = \begin{vmatrix} \sum_{r=1}^{n} 2r & x & n(n+1) \\ \sum_{r=1}^{n}(6r^2-1) & y & n^2(2n+3) \\ \sum_{r=1}^{n}(4r^3-2nr) & z & n^3(n+1) \end{vmatrix}$

$$= \begin{vmatrix} n(n+1) & x & n(n+1) \\ n^2(2n+3) & y & n^2(2n+3) \\ n^3(n+1) & z & n^3(n+1) \end{vmatrix} = 0$$

(as C_1 and C_3 are same)

15. (b):



A is the set of points on and above the line $y = 1$ ...(i) in the Argand plane. B is the set of points on the circle $(x - 2)^2 + (y - 1)^2 = 9$ ...(ii) and

$C = \text{Re}(1 - i)z = \text{Re}(1 - i)(x + iy)$

$\Rightarrow \ x + y = \sqrt{2}$ ... (iii)

Hence $A \cap B \cap C$ has only one point of intersection.

16. (d): $|z| - |w| < |z - w|$ and $|z - w|$ is the distance between z and w. Here, z is fixed. Hence distance between z and w would be maximum for diametrically opposite points. Therefore,

$|z - w| < 6 \quad \Rightarrow \quad -6 < |z| - |w| < 6$

$\Rightarrow \ -3 < |z| - |w| + 3 < 9$

17. (c): $\phi(1) = 1, \phi(p^n) = p^{n-1}(p - 1)$

$\phi(mn) = \phi(m) \cdot \phi(n)$

$\phi(8n + 4) = \phi(4(2n + 1)) = \phi(4) \cdot \phi(2n + 1)$

$= \phi(2)^2 \cdot \phi(2n + 1) = 2 \cdot \phi(2n + 1)$

18. (b): $\phi(n)$ is odd

$\Rightarrow \ \phi(p^n)$ is odd

$\Rightarrow \ p^{n-1}(p - 1)$ is odd.

$\because \ p$ is prime. The only value p can take is $p = 2$

$\therefore \ \phi(2^n)$ is odd.

$\Rightarrow \ 2^{n-1}(2 - 1) = 2^{n-1}$ is odd.

$\Rightarrow \ n - 1 = 0 \Rightarrow n = 1$

$\therefore \ \phi(1) = 1 = \phi(2)$

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Half of engineering seats lie vacant

Nagaland's only engineering institute doesn't have a single student, latest data shared by the All India Council for Technical Education (AICTE) show, as approved tech colleges in several states struggled with more than 50% seat vacancies in 2016-17.

Besides Nagaland's 100% vacancy for its 240 seats, the other states with a high low-enrolment record are Himachal Pradesh (75%), Haryana (72%) and Uttar Pradesh (64%).

The poor admission rate has prompted several states to ask the AICTE, the country's technical education regulator, to ban the move to create more seats in engineering colleges. This comes at a time when states such as Haryana, Madhya Pradesh, Chhattisgarh, Telangana, Maharashtra and Rajasthan have submitted plans to AICTE, which reports to the Union human resource development ministry, to set up new technical institutions.

"Many new engineering colleges are coming up every year. AICTE grants them approval if the infrastructure and faculty requirements are in place. But one needs to examine the demand and supply," said a ministry official, requesting anonymity.

Experts say the poor enrolment could be attributed to a near-stagnated job market for engineers or availability of a glut of career options for students from non-engineering fields.

"While the industry is moving towards automation, our institutes are still following obsolete syllabus. Our institutes need to do some soul-searching and align their teaching and curriculum with the requirement of the industry. Naturally, students will stop taking up certain courses if they are unable to find jobs," former AICTE chairman SS Mantha said on 8th March.

All these factors, including an adverse job market, have contributed to low campus placements and a decline in the students willing to study engineering.

The regulator is said to be taking a clutch of measures to ensure more seats are filled so that institutes do not have to apply for closure. Scholarship schemes, named Pragati and Saksham, are being floated to motivate and assist girls and differently-abled students.

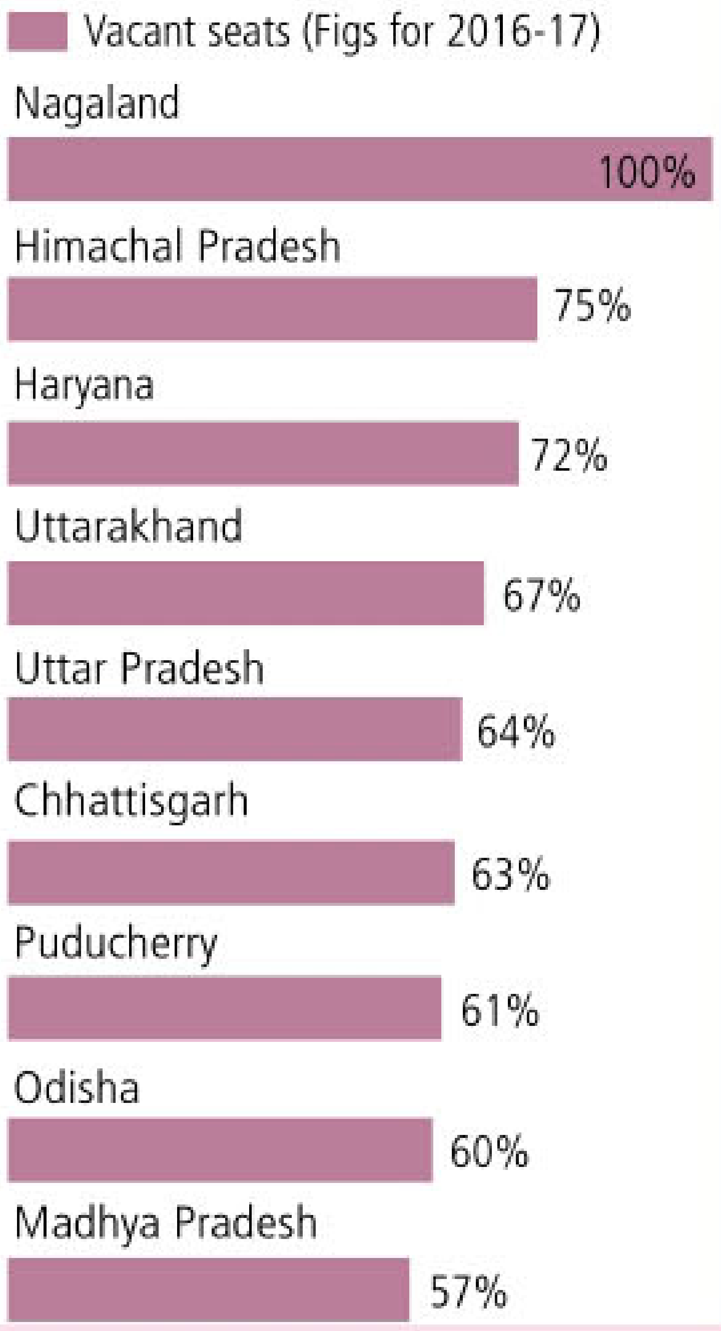
The AICTE is providing 10 supernumerary seats under the Jammu and Kashmir special scholarship programme. "Technical institutes have been granted permission to run vocational courses on their campuses," said another official, who doesn't wish to be named.

The regulator's data reveal that Manipur is a rare case as it has no vacancies at all. After Nagaland, Meghalaya has the biggest number of vacancies at 47% in the northeastern region, followed by Sikkim and Tripura with less than 40%. With 18 colleges, Assam's vacancy record stands at 34%.

◆◆

Losing charm

As 50% of total seats in India are vacant, experts hint at a near-stagnated job market



Courtesy : Hindustan Times

JEE MAIN

PRACTICE PAPER 2018

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1. If α, β are the roots of the equation $ax^2 + bx + c = 0$ and $S_n = \alpha^n + \beta^n$ (where $n \geq 2$) then $aS_{n+1} + bS_n + cS_{n-1} =$

(a) 0 (b) $a + b + c$

(c) $(a + b + (c)n$ (d) $n^2 abc$

2. If tan$A$, tan$B$ are the roots of $x^2 - Px + Q = 0$ (where $P, Q \in R$) then the value of $\sin^2(A + B) =$

(a) $\dfrac{P^2}{P^2+(1-Q)^2}$ (b) $\dfrac{P^2}{P^2+Q^2}$

(c) $\dfrac{Q^2}{P^2+(1-Q)^2}$ (d) $\dfrac{P^2}{(P+Q)^2}$

3. If $a_k = \dfrac{1}{k(k+1)}$, for $k = 1, 2, 3, \ldots\ldots, n$, then $\left(\sum_{k=1}^{n} a_k\right)^2 =$

(a) $\dfrac{n}{n+1}$ (b) $\dfrac{n^2}{(n+1)^2}$

(c) $\dfrac{n^4}{(n+1)^4}$ (d) $\dfrac{n^6}{(n+1)^6}$

4. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k!}\left(\sum_{n=1}^{k} 2^{n-1}\right) =$

(a) e (b) $e^2 + e$ (c) e^2 (d) $e^2 - e$

5. The sum of the series $\dfrac{x}{1-x^2} + \dfrac{x^2}{1-x^4} + \dfrac{x^4}{1-x^8} + \ldots\ldots$ to infinite terms, if $|x| < 1$ is

(a) $\dfrac{x}{1-x}$ (b) $\dfrac{1}{1-x}$ (c) $\dfrac{1+x}{1-x}$ (d) 1

6. The ratio of sum of first three terms of a G.P. to the sum of first six terms is 64:91 ,the common ratio of G.P. is

(a) $\dfrac{3}{2}$ (b) $\dfrac{3}{4}$ (c) $\dfrac{4}{3}$ (d) $\dfrac{2}{3}$

7. If 1, $\log_{81}(3^x + 48), \log_9\left(3^x - \dfrac{8}{3}\right)$ are in A.P., then the value of x equals

(a) 9 (b) 6 (c) 2 (d) 4

8. The number of positive integral solutions of the equation $\begin{vmatrix} y^3+1 & y^2z & y^2x \\ yz^2 & z^3+1 & z^2x \\ yx^2 & x^2z & x^3+1 \end{vmatrix} = 11$ is

(a) 1 (b) 2 (c) 3 (d) 4

9. If $a - 2b + c = 1$, then the value of $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$ is

(a) x (b) $-x$ (c) -1 (d) 1

10. If $a = \cos\dfrac{4\pi}{3} + i\sin\dfrac{4\pi}{3}$, then $\begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix}$ is

(a) purely real (b) purely imaginary

(c) $a\left(\cos\dfrac{2\pi}{3} + i\sin\dfrac{2\pi}{3}\right)$ (d) a

11. $\begin{vmatrix} {}^xC_r & {}^xC_{r+1} & {}^xC_{r+2} \\ {}^yC_r & {}^yC_{r+1} & {}^yC_{r+2} \\ {}^zC_r & {}^zC_{r+1} & {}^zC_{r+2} \end{vmatrix} - \begin{vmatrix} {}^xC_r & {}^{x+1}C_{r+1} & {}^{x+2}C_{r+2} \\ {}^yC_r & {}^{y+1}C_{r+1} & {}^{y+2}C_{r+2} \\ {}^zC_r & {}^{z+1}C_{r+1} & {}^{z+2}C_{r+2} \end{vmatrix} =$

(a) 0 (b) 2^n

(c) ${}^{x+y+z}C_r$ (d) ${}^{x+y+z}C_{r+2}$

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12. If A and B are square matrices of the same order and A is non-singular, then for a positive integer n, $(A^{-1}BA)^n$ is equal to

(a) $A^{-n}B^nA^n$ (b) $A^nB^nA^{-n}$
(c) $A^{-1}B^nA$ (d) $n(A^{-1}BA)$

13. If A and B are any two different square matrices of order n with $A - B$ is non-singular and $A^3 = B^3$ and $A(AB) = B(BA)$, then

(a) $A^2 + B^2 = O$ (b) $A^2 + B^2 = I$
(c) $A^2 + B^3 = I$ (d) $A^3 + B^3 = O$

14. If $x = \sin\frac{2\pi}{7} + \sin\frac{4\pi}{7} + \sin\frac{8\pi}{7}$ and

$y = \cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{8\pi}{7}$, then $x^2 + y^2 =$

(a) 1 (b) 2 (c) 3 (d) 4

15. The number of values of y in $[-2\pi, 2\pi]$ satisfying the equation $|\sin 2x| + |\cos 2x| = |\sin y|$ is

(a) 1 (b) 2 (c) 3 (d) 4

16. Let $\theta \in [0, 4\pi]$ satisfying the equation $(\sin\theta + 2)(\sin\theta + 3)(\sin\theta + 4) = 6$. If the sum of all values of θ is $K\pi$, then value of K is

(a) 6 (b) 5 (c) 4 (d) 2

17. The line $x + y = 1$ meets x-axis at A and y-axis at B. P is the mid-point of AB. P_1 is the foot of the perpendicular from P to OA; M_1 is that from P_1 to OP; P_2 is that from M_1 to OA; M_2 is that from P_2 to OP; P_3 is that from M_2 to OA and so on. If P_n denotes the n^{th} foot of the perpendicular on OA from M_{n-1}, then OP_n is

(a) $\frac{1}{2}$ (b) $\frac{1}{2^n}$ (c) $\frac{1}{2^{n/2}}$ (d) $\frac{1}{\sqrt{2}}$

18. The orthocentre of the triangle formed by the lines $x + y = 1$, $2x + 3y = 6$ and $4x - y + 4 = 0$ lies in

(a) I quadrant (b) II quadrant
(c) III quadrant (d) IV quadrant

19. $f: R \to R, f(x) = x|x|$ is

(a) one-one but not onto
(b) onto but not one-one
(c) Both one-one and onto
(d) neither one-one nor onto

20. The domain of the function

$$f(x) = \sin^{-1}\left(\log_2\left(\frac{x^2}{2}\right)\right) \text{is}$$

(a) $[-2, 2]$ (b) $[-2, -1]$
(c) $[1, 2]$ (d) $[-2, -1] \cup [1, 2]$

21. For each positive integer n, let

$S_n = \frac{3}{1.2.4} + \frac{4}{2.3.5} + \frac{5}{3.4.6} + ... + \frac{n+2}{n(n+1)(n+3)}$. Then $\lim_{n\to\infty} s_n$ equals

(a) $\frac{29}{6}$ (b) $\frac{29}{36}$ (c) 0 (d) $\frac{29}{18}$

22. $\lim_{x\to 0} \frac{a^{\tan x} - a^{\sin x}}{\tan x - \sin x}$ is equal to $(a > 0)$

(a) $\log_e a$ (b) 1 (c) 0 (d) ∞

23. Let $f: R \to R$ be a differentiable function satisfying $f(y)f(x - y) = f(x)\ \forall\, x, y \in R$ and $f'(0) = p$, $f'(5) = q$, then $f(5)$ is

(a) p^2/q (b) p/q (c) q/p (d) q

24. $f(x) = \begin{cases} \left(\tan\left(\frac{\pi}{4} + x\right)\right)^{\frac{1}{x}} & \text{if } x \neq 0 \\ \lambda & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$, then value of λ is

(a) 1 (b) e (c) e^2 (d) 0

25. Let $f(x) = \begin{cases} x^n \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$

then $f(x)$ is continuous but not differentiable at $x = 0$, if

(a) $n \in (0, 1]$ (b) $n \in [1, \infty)$
(c) $n \in (-\infty, 0)$ (d) $n = 0$

26. If $2p^2 - 3q^2 + 4pq - p = 0$ and a variable line $px + qy = 1$ always touches a parabola whose axis is parallel to X-axis, then equation of the parabola is

(a) $(y - 4)^2 = 24(x - 2)$ (b) $(y - 3)^2 = 12(x - 1)$
(c) $(y - 4)^2 = 12(x - 2)$ (d) $(y - 2)^3 = 24(x - 4)$

27. The locus of point of intersection of tangents to the parabola $y^2 = 4ax$, the angle between them being always 45°, is

(a) $x^2 - y^2 + 6ax - a^2 = 0$
(b) $x^2 - y^2 - 6ax + a^2 = 0$
(c) $x^2 - y^2 + 6ax + a^2 = 0$
(d) $x^2 - y^2 - 6ax - a^2 = 0$

28. If z_1, z_2, z_3 and z_4 be the consecutive vertices of a square, then $z_1^2 + z_2^2 + z_3^2 + z_4^2$ equals

(a) $z_1z_2 + z_2z_3 + z_3z_4 + z_4z_1$
(b) $z_1z_2 + z_1z_3 + z_1z_4 + z_2z_3 + z_2z_4 + z_3z_4$
(c) 0 (d) None of these

29. If z is a complex number satisfying $z^4 + z^3 + 2z^2 + z + 1 = 0$, then the set of possible values of $|z|$ is
(a) $\{1, 2\}$ (b) $\{1\}$ (c) $\{1, 2, 3\}$ (d) $\{1, 2, 3, 4\}$

30. Let A, B and C represents the complex numbers z_1, z_2 and z_3 in the argand plane. If circumcentre of the triangle ABC is at the origin, then the complex number corresponding to orthocentre is

(a) $\frac{1}{4}(z_1 + z_2 + z_3)$ (b) $\frac{1}{3}(z_1 + z_2 + z_3)$

(c) $\frac{1}{2}(z_1 + z_2 + z_3)$ (d) $z_1 + z_2 + z_3$

SOLUTIONS

1. (a) : $S_{n+1} = \alpha^{n+1} + \beta^{n+1}$

$$= (\alpha + \beta)(\alpha^n + \beta^n) - \alpha\beta(\alpha^{n-1} + \beta^{n-1}) = -\frac{b}{a}\cdot S_n - \frac{c}{a}\cdot S_{n-1}$$

2. (a) : $\tan(A+B) = \frac{P}{1-Q}$

then $\sin^2(A+B) = \frac{\tan^2(A+B)}{1+\tan^2(A+B)}$

3. (b) : $\left(\sum_{k=1}^{n} a_k\right)^2 = \left(\frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \ldots\ldots + \frac{1}{n} - \frac{1}{n+1}\right)^2$

$$= \frac{n^2}{(n+1)^2}$$

4. (d) : $\sum_{k=1}^{\infty} \frac{1}{k!}(1 + 2 + 2^2 + \ldots\ldots + 2^{k-1})$

$$= \sum_{k=1}^{\infty} \frac{2^k - 1}{k!} = e^2 - e$$

5. (a) : The general term of the series is $t_n = \frac{x^{2^{n-1}}}{1 - x^{2^n}}$

$$= \frac{1 + x^{2^{n-1}} - 1}{\left(1 + x^{2^{n-1}}\right)\left(1 - x^{2^{n-1}}\right)}$$

$$\therefore \quad t_n = \frac{1}{1 - x^{2^{n-1}}} - \frac{1}{1 - x^{2^n}}$$

Now, $S_n = \sum_{n=1}^{n} t_n = \left[\left\{\frac{1}{1-x} - \frac{1}{1-x^2}\right\}\right.$

$$\left. + \left\{\frac{1}{1-x^2} - \frac{1}{1-x^4}\right\} + \ldots + \left\{\frac{1}{1-x^{2^{n-1}}} - \frac{1}{1-x^{2^n}}\right\}\right]$$

$$= \frac{1}{1-x} - \frac{1}{1-x^{2^n}}$$

∴ The sum to infinite terms

$$= \lim_{n\to\infty} S_n = \frac{1}{1-x} - 1 = \frac{x}{1-x} \left[\because \lim_{n\to\infty} x^{2^n} = 0, \text{ as } |x| < 1\right]$$

6. (b) : Given $\frac{S_3}{S_6} = \frac{64}{91} = \frac{a(r^3-1)}{a(r^6-1)}$

$$\Rightarrow \frac{(r^3-1)}{(r^3+1)(r^3-1)} = \frac{64}{91} \quad \Rightarrow \quad r^3 = \frac{27}{64} \quad \therefore \quad r = \frac{3}{4}$$

7. (c) : Given 1, $\log_{9^2}(3^x + 48)$, $\log_9(3^x - 8/3) \in$ A.P.

$$\Rightarrow \log_9 9, \frac{1}{2}\log_9(3^x + 48), \log_9(3^x - 8/3) \in \text{A.P.}$$

$\Rightarrow$ 9, $(3^x + 48)^{1/2}$, $3^x - 8/3 \in$ G.P.
[$\because$ $\log a$, $\log b$, $\log c \in$ A.P. $\Rightarrow$ a, b, $c \in$ G.P.]
$\therefore$ $3^x + 48 = 9(3^x - 8/3)$
$\Rightarrow$ $8.3^x = 72 \Rightarrow 3^x = 9 \Rightarrow 3^x = 3^2 \Rightarrow x = 2$.

8. (c) : Multiply by y, z and x in rows 1,2 and 3 respectively and then take common y, z and x from column 1, 2 and 3 respectively, then

$$\begin{vmatrix} y^3+1 & y^3 & y^3 \\ z^3 & z^3+1 & z^3 \\ x^3 & x^3 & x^3+1 \end{vmatrix} = 11$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & y^3 \\ -1 & 1 & z^3 \\ 0 & -1 & x^3+1 \end{vmatrix} = 11 \ (C_1 \to C_1 - C_2 \text{ and } C_2 \to C_2 - C_3)$$

$\Rightarrow$ $1(x^3 + 1 + z^3) + y^3(1) = 11 \Rightarrow x^3 + y^3 + z^3 = 10$
So, solutions are (1, 1, 2), (1, 2, 1) or (2, 1, 1)

9. (c) : $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$

Applying the operation, $R_1 \to R_1 - 2R_2 + R_3$ and $R_3 \to R_3 - R_2$, and using $a - 2b + c = 1$ the determinant reduces to

$$\begin{vmatrix} 0 & 0 & 1 \\ x+2 & x+3 & x+b \\ 1 & 1 & c-b \end{vmatrix} = -1$$

10. (b) : $a = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3} = \omega^2$

$$\therefore \begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{vmatrix} = 3(\omega - \omega^2), \text{which is purely imaginary.}$$

11. (a) :

$$\begin{vmatrix} {}^xC_r & {}^xC_{r+1} & {}^xC_{r+2} \\ {}^yC_r & {}^yC_{r+1} & {}^yC_{r+2} \\ {}^zC_r & {}^zC_{r+1} & {}^zC_{r+2} \end{vmatrix} = \begin{vmatrix} {}^xC_r & {}^{x+1}C_{r+1} & {}^{x+1}C_{r+2} \\ {}^yC_r & {}^{y+1}C_{r+1} & {}^{y+1}C_{r+2} \\ {}^zC_r & {}^{z+1}C_{r+1} & {}^{z+1}C_{r+2} \end{vmatrix}$$

[By applying $C_2 \to C_2 + C_1$, $C_3 \to C_3 + C_2$]

Now, apply $C_3 \to C_3 + C_2$, $\begin{vmatrix} {}^xC_r & {}^{x+1}C_{r+1} & {}^{x+2}C_{r+2} \\ {}^yC_r & {}^{y+1}C_{r+1} & {}^{y+2}C_{r+2} \\ {}^zC_r & {}^{z+1}C_{r+1} & {}^{z+2}C_{r+2} \end{vmatrix}$

$\therefore$ Required result = 0

12. (c) : $(A^{-1}BA)^2 = (A^{-1}BA)(A^{-1}BA)$

$= A^{-1}B(AA^{-1})BA = A^{-1}BIBA = A^{-1}B^2A$

$\Rightarrow (A^{-1}BA)^3 = (A^{-1}B^2A)(A^{-1}BA) = A^{-1}B^2(AA^{-1})BA$

$= A^{-1}B^2IBA = A^{-1}B^3A$ and so on

$\Rightarrow (A^{-1}BA)^n = A^{-1}B^nA$

13. (a) : We have, $A^3 = B^3$ ...(i)

and $A^2B = B^2A$ ...(ii)

Subtracting (ii) from (i), we get

$(A^2 + B^2)(A - B) = O$

$\because |A - B| \neq 0$

$\therefore A^2 + B^2 = O$

14. (b) : $x^2 + y^2 = 3 + 2\left(\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}\right)$

$= 3 + 2\left(\frac{-1}{2}\right) = 2$

15. (d) : $1 \le |\sin 2x| + |\cos 2x| \le \sqrt{2}$

and $|\sin y| \le 1 \Rightarrow \sin y = \pm 1 \Rightarrow y = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}$

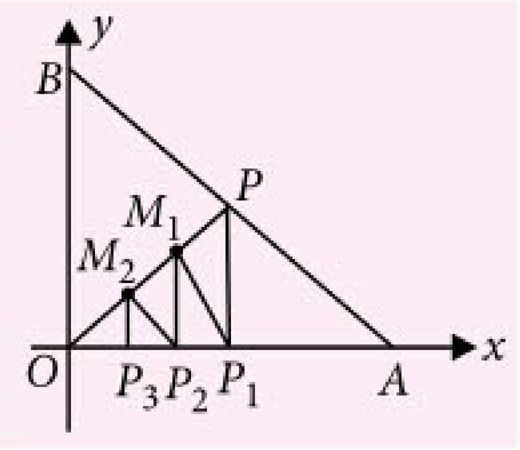
16. (b) : $\sin\theta = -1 \Rightarrow \theta = \frac{3\pi}{2}, \frac{7\pi}{2}$

$\therefore K = 5$

17. (b) : $x + y = 1$ meets x-axis at $A(1, 0)$ and y-axis at $B(0, 1)$. The coordinates of P are $(1/2, 1/2)$ and PP_1 is perpendicular to OA.

$\Rightarrow OP_1 = P_1P = 1/2$

Equation of line OP is $y = x$



We have

$(OM_{n-1})^2 = (OP_n)^2 + (P_nM_{n-1})^2 = 2(OP_n)^2 = 2p_n^2$ (say)

Also, $(OP_{n-1})^2 = (OM_{n-1})^2 + (P_{n-1}M_{n-1})^2$

$\Rightarrow (OP_{n-1})^2 = 2p_n^2 + \frac{1}{2}p_{n-1}^2$

$\Rightarrow p_n^2 = \frac{1}{4}p_{n-1}^2 \Rightarrow p_n = \frac{1}{2}p_{n-1}$

$\therefore OP_n = p_n = \frac{1}{2}p_{n-1} = \frac{1}{2^2}p_{n-2} = ... = \frac{1}{2^{n-1}}p_1 = \frac{1}{2^n}$

18. (a) : Coordinates of A and B are $(-3, 4)$ and $\left(-\frac{3}{5}, \frac{8}{5}\right)$ respectively. Let orthocentre is $P(h, k)$.

Then, (slope of PA) × (slope of BC) = –1

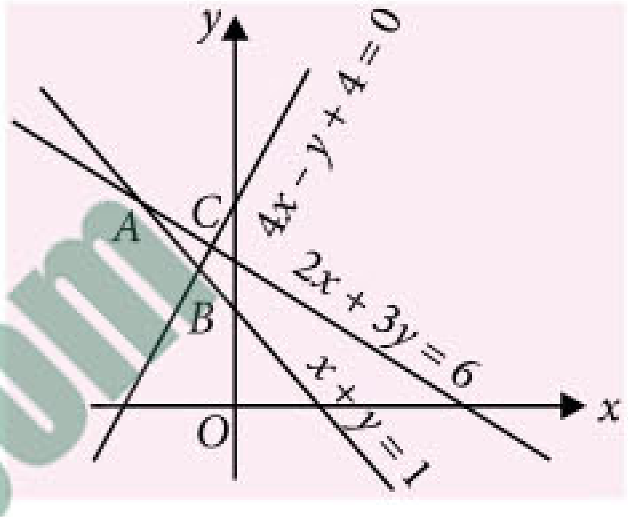
$\Rightarrow \frac{k-4}{h+3} \times 4 = -1$

$\Rightarrow 4k - 16 = -h - 3$

$\Rightarrow h + 4k = 13$ .. (i)

and slope of PB × slope of AC = – 1

$\Rightarrow \frac{k - \frac{8}{5}}{h + \frac{3}{5}} \times \left(-\frac{2}{3}\right) = -1$



$\Rightarrow 10k - 16 = 15h + 9$

$\Rightarrow 15h - 10k + 25 = 0 \Rightarrow 3h - 2k + 5 = 0$ ... (ii)

Solving (i) and (ii), we get $h = \frac{3}{7}, k = \frac{22}{7}$

Hence, orthocentre lies in I quadrant.

19. (c) : Given that $f(x) = \begin{cases} x^2 \text{ if } x > 0 \\ 0 \text{ if } x = 0 \\ -x^2 \text{ if } x < 0 \end{cases}$

Thus, $f(x)$ is both one-one and onto.

20. (d) : $f(x) = \sin^{-1}\left(\log_2\left(\frac{x^2}{2}\right)\right) \in \mathbf{R}$

$\Leftrightarrow -1 \le \log_2\left(\frac{x^2}{2}\right) \le 1 \Leftrightarrow \frac{1}{2} \le \frac{x^2}{2} \le 2 \Leftrightarrow 1 \le x^2 \le 4$

$\Leftrightarrow x \in [-2, -1] \cup [1, 2]$

21. (b) : Let $u_k = \frac{k+2}{k(k+1)(k+3)}$

$= \frac{(k+2)^2}{k(k+1)(k+2)(k+3)} = \frac{k^2+4k+4}{k(k+1)(k+2)(k+3)}$

$= \frac{k(k+1)+3k+4}{k(k+1)(k+2)(k+3)}$

$= \frac{1}{(k+2)(k+3)} + \frac{3}{(k+1)(k+2)(k+3)} + \frac{4}{k(k+1)(k+2)(k+3)}$

$= \left(\frac{1}{k+2} - \frac{1}{k+3}\right) - \frac{3}{2}\left[\frac{1}{(k+2)(k+3)} - \frac{1}{(k+1)(k+2)}\right]$

$$-\frac{4}{3}\left[\frac{1}{(k+1)(k+2)(k+3)}-\frac{1}{k(k+1)(k+2)}\right]$$

Now, put $k = 1, 2, 3, \ldots .n$ and add. Thus

$S_n = u_1 + u_2 + \ldots .. u_n$

$$=\left(\frac{1}{3}-\frac{1}{n+3}\right)-\frac{3}{2}\left[\frac{1}{(n+2)(n+3)}-\frac{1}{2.3}\right]$$

$$-\frac{4}{3}\left[\frac{1}{(n+1)(n+2)(n+3)}-\frac{1}{1.2.3}\right]$$

Therefore, $\lim\limits_{n\to\infty} S_n=\frac{1}{3}+\frac{3}{12}+\frac{4}{18}=\frac{29}{36}$

22. (a) : We have,

$$\lim_{x\to 0}\frac{a^{\tan x}-a^{\sin x}}{\tan x-\sin x}=\lim_{x\to 0} a^{\sin x}\left(\frac{a^{\tan x-\sin x}-1}{\tan x-\sin x}\right)$$

$$=\lim_{x\to 0}(a^{\sin x})\times\lim_{t\to 0}\left(\frac{a^t-1}{t}\right)(\text{where } t=\tan x-\sin x)$$

$= a^0 \times \log_e a = \log_e a$

23. (c) : $y=0 \Rightarrow f(0)=1$ and $x=0 \Rightarrow f(-y)=\frac{1}{f(y)}$

Hence, $f(x + y) = f(x)f(y)$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=f(x)\lim_{h\to 0}\frac{f(h)-1}{h}=f(x).f'(0)=pf(x)$$

Put $x=5, f(5)=\frac{q}{p}$

24. (c) : $\lambda=\lim\limits_{x\to 0}\left(\frac{1+\tan x}{1-\tan x}\right)^{\frac{1}{x}}=e^2$

25. (a) : R.H.L. $=\lim\limits_{x\to 0^+} f(x)$

$$=\lim_{h\to 0} f(0+h) = \lim_{h\to 0} h^n\cdot\sin\left(\frac{1}{h}\right) = 0^n\cdot(-1 \text{ to } 1)$$

$\because\ f(0)=0$

$\therefore\ n > 0$ ...(i)

$$Rf'(0)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$$

$$=\lim_{h\to 0}\frac{h^n\sin\left(\frac{1}{h}\right)-0}{h} = \lim_{h\to 0} h^{n-1}\sin\left(\frac{1}{h}\right)=0^{n-1}(-1\text{ to }1)$$

For not differentiable, $n - 1 \le 0 \Rightarrow n \le 1$ ...(ii)

From (i) & (ii), $0 < n \le 1$

26. (c) : The parabola be $(y - a)^2 = 4b\,(x - c)$

Equation of tangent is $(y-a)=-\frac{p}{q}(x-c)-\frac{bq}{p}$

Comparing with $px + qy = 1$, we get

$cp^2 - bq^2 + apq - p = 0$

$\therefore\ \frac{c}{2}=\frac{b}{3}=\frac{a}{4}=1$

Thus, equation of parabola is $(y - 4)^2 = 12(x - 2)$

27. (c) : Equation of tangent is $y=mx+\frac{a}{m}$

$$\Rightarrow m^2x-my+a=0\Rightarrow m_1+m_2=\frac{y}{x}, m_1m_2=\frac{a}{x}$$

$$\tan 45^\circ=\left|\frac{m_1-m_2}{1+m_1m_2}\right|\Rightarrow\left(\frac{y}{x}\right)^2-4\left(\frac{a}{x}\right)=\left(1+\frac{a}{x}\right)^2$$

$\Rightarrow\ x^2 - y^2 + 6ax + a^2 = 0$

28. (a) : We know that $\frac{z_2-z_1}{z_4-z_1}=\frac{|z_2-z_1|}{|z_4-z_1|}e^{i\pi/2}=i$

(as $|z_2 - z_1| = |z_4 - z_1|$)

$\Rightarrow\ (z_4-z_1)^2+(z_2-z_1)^2=0$ ...(i)

Similarly, $\frac{z_4-z_3}{z_2-z_3}=i$

$\Rightarrow\ (z_4-z_3)^2+(z_2-z_3)^2=0$ ...(ii)

On adding (i) and (ii), we get

$$2\left[z_1^2+z_2^2+z_3^2+z_4^2-z_1z_2-z_4z_1-z_4z_3-z_2z_3\right]=0$$

$$\Rightarrow\ z_1^2+z_2^2+z_3^2+z_4^2=z_1z_2+z_2z_3+z_3z_4+z_4z_1$$

29. (b) : The given equation is $(z^2 + z + 1)(z^2 + 1) = 0$.
$z = \pm i$, ω, ω^2, ω being an imaginary cube root of unity. Thus $|z| = 1$.

30. (d) : Centroid of ΔABC is at $\frac{z_1+z_2+z_3}{3}$.

Orthocentre divides centroid and circumcentre in the ratio 2 : 3 externally.

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MPP CLASS XII ANSWER KEY

1. (b)	2. (a)	3. (a)	4. (a)	5. (c)
6. (a)	7. (a,b)	8. (b,c)	9. (b)	10. (a,c)
11. (a,d)	12. (a,d)	13. (b)	14. (a)	15. (c)
16. (c)	17. (0)	18. (9)	19. (6)	20. (2)

YOU ASK WE ANSWER

Do you have a question that you just can't get answered?
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. Find the projection of the line $\frac{x-1}{0}=\frac{y-3}{1}=\frac{z-4}{7}$ in the plane $2x - y + z + 3 = 0$.

(N.Subramanjan, Chennai)

Ans. Given line is $\frac{x-1}{0}=\frac{y-3}{1}=\frac{z-4}{7}=\lambda$ (say).

Coordinate of general point on the line is $(1, \lambda + 3, 7\lambda + 4)$.
Its foot on the plane $2x - y + z + 3 = 0$ is

$$\frac{x-1}{2}=\frac{y-\lambda-3}{-1}=\frac{z-7\lambda-4}{1}=\frac{-(2-\lambda-3+7\lambda+4+3)}{4+1+1}$$
$$= -(\lambda + 1)$$

$\Rightarrow$ $x - 1 = -2\lambda - 2, y - 4 = 2\lambda, z = 6\lambda + 3$

$\Rightarrow$ $x + 1 = -2\lambda, y - 4 = 2\lambda, z - 3 = 6\lambda$

$\Rightarrow$ $\frac{(x+1)}{-1}=\frac{y-4}{1}=\frac{z-3}{3}$, which is the projection of the line on given plane.

2. Check differentiability of the function

$$f(x) = (x^2 - 1)\,|x^2 - 3x + 2| + \cos|x|.$$

(Saksham, Delhi)

Ans. The function

$f(x) = (x^2 - 1)|x^2 - 3x + 2| + \cos|x|$
$= (x^2 - 1)|(x - 1)(x - 2)| + \cos x$ $[\because \cos(-x) = \cos x]$

$$\Rightarrow\ f(x)=\begin{cases}-(x^2-1)(x-1)(x-2)+\cos x, & 1\le x\le 2\\ (x^2-1)(x-1)(x-2)+\cos x, & \text{otherwise.}\end{cases}$$

Thus, we need to check differentiability only at $x = 1, 2$.
At $x = 1$, we have

$$f'(1^+)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$

$$=\lim_{h\to 0}\frac{\left[\begin{array}{r}\{1-(1+h)^2\}(1+h-1)(1+h-2)\\ +\cos(1+h)\end{array}\right]-\cos 1}{h}$$

$$=\lim_{h\to 0}\frac{(h^2+2h)h(1-h)}{h}+\frac{\cos(1+h)-\cos 1}{h}$$

$$=\lim_{h\to 0}(h^2+2h)(1-h)-\frac{\sin(1+h)}{1}=-\sin 1$$

$$f'(1^-)=\lim_{h\to 0}\frac{f(1)-f(1-h)}{h}$$

$$=\lim_{h\to 0}\frac{\cos 1-\left[\begin{array}{r}\{(1-h)^2-1\}(1-h-1)(1-h-2)\\ +\cos(1-h)\end{array}\right]}{h}$$

$$=\lim_{h\to 0}\frac{-(h^2-2h)h(1+h)}{h}+\frac{\cos 1-\cos(1-h)}{h}$$

$$=\lim_{h\to 0}-(h^2-2h)(1+h)-\frac{\sin(1-h)}{h}=-\sin 1.$$

Thus, f is differentiable at $x = 1$.
At $x = 2$, we have

$$f'(2^+)=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$$

$$=\lim_{h\to 0}\frac{\left[\begin{array}{r}\{(2+h)^2-1\}(2+h-1)(2+h-2)\\ +\cos(2+h)\end{array}\right]-\cos 2}{h}$$

$$=\lim_{h\to 0}\frac{(h^2+4h+3)(1+h)h}{h}+\frac{\cos(2+h)-\cos 2}{h}$$

$$=\lim_{h\to 0}(h^2+4h+3)(1+h)+\frac{-\sin(2+h)}{1}=3-\sin 2.$$

$$f'(2^-)=\lim_{h\to 0}\frac{f(2)-f(2-h)}{h}$$

$$=\lim_{h\to 0}\frac{\cos 2-\left[\begin{array}{r}\{1-(2-h)^2\}(2-h-1)(2-h-2)\\ +\cos(2-h)\end{array}\right]}{h}$$

$$=\lim_{h\to 0}\frac{-(h^2-4h+3)h(1-h)}{h}+\frac{\cos 2-\cos(2-h)}{h}$$

$$=\lim_{h\to 0}-(h^2-4h+3)(1-h)+\frac{-\sin(2-h)}{1}=-3-\sin 2.$$

Thus, f is not differentiable at $x = 2$.
Hence f is differentiable on $x \in R \sim \{2\}$.

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JEE WORK CUTS

PAPER-1

SECTION-I

Integer Answer Type

This section contains eight questions. The answer to each question is a single digit integer ranging from 0 to 9, both inclusive. For each question, darken the bubble corresponding to the correct integer in the ORS. Marking scheme : +4 If the bubble corresponding to the answer is darkened, 0 In all other cases.

1. Let $f: R \to R$ be a differentiable function satisfying $f(x) = f(y)\, f(x-y)\ \forall\ x, y \in R$ and $f'(0) = \int_0^4 \{2x\}\, dx$, where $\{.\}$ denotes the fractional part function and $f'(-3) = \alpha e^{\beta}$. Then, $|\alpha + \beta|$ is equal to ______.

2. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors, then find the maximum value of $|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2$

3. If $\int \frac{\ln x}{x^3}\, dx = -\frac{\ln x}{ax^2} - \frac{1}{bx^2} + c$, then the value of $a + b$ must be ______.

4. TP and TQ are any two tangents to a parabola and the tangent at a third point R cuts them in P' and Q' respectively, then the value of $\frac{TP'}{TP} + \frac{TQ'}{TQ}$ must be ____.

5. If $\alpha, \beta, \gamma, \delta$ are the solutions of the equation $\tan\left(\theta + \frac{\pi}{4}\right) = 3\tan 3\theta$, no two of which have equal tangents, then the value of $\tan\alpha + \tan\beta + \tan\gamma + \tan\delta$ is equal to ______.

6. The number of solutions for the equation $\log_4(2x^2 + x + 1) - \log_2(2x - 1) = 1$, is ______.

7. Find the value of

$$4\left(\cos\frac{2\pi}{15} + \cos\frac{4\pi}{15} - \cos\frac{7\pi}{15} - \cos\frac{\pi}{15}\right)$$

8. If $(y^2 - 5y + 3)(x^2 + x + 1) < 2x$ for all $x \in R$, then number of integral values of y will be

SECTION-II

One or More Than One Option Correct Type

This section contains ten questions. Each question has four options (a), (b), (c) and (d). One or More than one of these four option (s) is (are) correct. For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS. Marking scheme : +4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened, 0 If none of the bubbles is darkened, −2 In all other cases.

9. If a, b, c are real numbers and z is a complex number such that $a^2 + b^2 + c^2 = 1$ and $b + ic = (1 + a)z$, then $\frac{1+iz}{1-iz}$ equals

(a) $\frac{b-ic}{1-ia}$ (b) $\frac{a+ib}{1+c}$ (c) $\frac{1-c}{a-ib}$ (d) $\frac{1+a}{b+ic}$

10. If the quadratic polynomial $P(x) = (p-3)x^2 - 2px + 3p - 6$ ranges from $[0, \infty)$ for every $x \in R$, then the value of p can be

(a) $\frac{3}{2}$ (b) 4 (c) 6 (d) 7

11. All non-zero complex numbers on the complex plane satisfying $z + \frac{1}{z} \in R$ can lie on

(a) $y = 0$ (b) $x = 0$

(c) $y = x$

(d) a unit circle with centre at the origin

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12. Find the domain of $f(x) = \sin^{-1}[2 - 4x^2]$ ([.] denotes the greatest integer function)

(a) $\left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right] - \{0\}$ (b) $\left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right]$

(c) $\left[-\frac{\sqrt{3}}{2}, 0\right)$ (d) $\left[-\frac{\sqrt{3}}{2}, 8\right]$

13. $\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j} 1$ is equal to

(a) $\frac{n(n+1)(n+2)}{6}$ (b) $\sum_{n=1}^{n} n^3$

(c) $\frac{n(n+1)(2n+1)}{6}$ (d) ${}^{n+2}C_3$

14. The line $3x + 6y = k$ intersect the curve $2x^2 + 2xy + 3y^2 = 1$ at points A and B. The circle on AB as diameter passes through the origin. The possible value(s) of k is (are)

(a) 3 (b) 4 (c) –4 (d) –3

15. Let $P(x) = x^2 + bx + c$, where b and c are integers. If $P(x)$ is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$, then

(a) $P(x) = 0$ has imaginary roots
(b) $P(x) = 0$ has roots of opposite sign
(c) $P(1) = 4$ (d) $P(1) = 6$

16.

$$\left[({}^nC_0 + {}^nC_3 + ...) - \frac{1}{2}({}^nC_1 + {}^nC_2 + {}^nC_4 + {}^nC_5 + ...)\right]^2 + \frac{3}{4}({}^nC_1 - {}^nC_2 + {}^nC_4 - {}^nC_5 + ...)^2 =$$

(a) 3 (b) 4 (c) 2 (d) 1

17. For every pair of continuous functions $f, g : [0, 1] \to R$, such that $\max\{f(x) : x \in [0, 1]\} = \max\{g(x) : x \in [0, 1]\}$, the correct statement(s) is (are):

(a) $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0, 1]$
(b) $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0, 1]$
(c) $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$ for some $c \in [0, 1]$
(d) $(f(c))^2 = (g(c))^2$ for some $c \in [0, 1]$

18. Let $\overrightarrow{PR} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\overrightarrow{SQ} = \hat{i} - 3\hat{j} - 4\hat{k}$ determine diagonals of a parallelogram $PQRS$ and $\overrightarrow{PT} = \hat{i} + 2\hat{j} + 3\hat{k}$ be another vector. Then the volume of the parallelepiped determined by the vectors $\overrightarrow{PT}, \overrightarrow{PQ}$ and $\overrightarrow{PS}$ is

(a) 5 (b) 20 (c) 10 (d) 30

SECTION-III

Matrix Match Type

This section contains 2 questions. Each question contains two columns, Column I and Column II. Column I has four entries (A), (B), (C) and (D). Column II has four entries (p), (q), (r) and (s). Match the entries in Column I with the entries in Column II. One or more entries in Column I may match with one or more entries in Column II. The ORS contains a 4 × 5 matrix whose layout will be similar to the one shown. For each entry in Column I, darken the bubbles of all the matching entries. For example, if entry (A) in Column I matches with entries (q), (r) and (s), then darken these three bubbles in the ORS. Similarly, for entries (B), (C) and (D). Marking scheme : For each entry in Column I. +2 If only the bubble(s) corresponding to all the correct match(es) is (are) darkened, 0 If none of the bubbles is darkened, –1 In all other cases.

	p	q	r	s
A	●	ⓠ	ⓡ	●
B	ⓟ	●	●	ⓢ
C	●	●	ⓡ	ⓢ
D	ⓟ	ⓠ	ⓡ	●

19. Match the following: -

Column-I	Column-II
(A) Consider the differential equation, $y = x\frac{dy}{dx} + \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ It's degree is	(p) 1
(B) The differential equation representing the family of curve $y^2 = 2c(x + \sqrt{c})$, where c is a parameter, is of degree	(q) 4
(C) Consider differential equation, $1 + \left(\frac{dy}{dx}\right) + \frac{1}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{1}{3!}\left(\frac{dy}{dx}\right)^3 + ...\infty = y$ then its degree is	(r) 2
(D) Consider the differential equation, $\frac{dy}{dx} = \left(1 + \left(\frac{d^2y}{dx^2}\right)^2\right)^{2/3}$ Its degree is	(s) 3

20. Consider the system of equations
$a_1x + b_1y + c_1z = d_1$
$a_2x + b_2y + c_2z = d_2$
$a_3x + b_3y + c_3z = d_3$

Now, let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $\Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$

$\Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$ and $\Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$

According to Crammer's rule,

$$x = \frac{\Delta_1}{\Delta}, y = \frac{\Delta_2}{\Delta}, z = \frac{\Delta_3}{\Delta}$$

Match the following: -

	Column-I		**Column-II**
(A)	If $\Delta \neq 0$ and $\Delta_1 = \Delta_2 = \Delta_3 = 0$, then the solutions are	(p)	Unique
(B)	If $\Delta_1 = \Delta_2 = \Delta_3 = \Delta = 0$, then the solutions are	(q)	Inconsistent
(C)	If $\Delta \neq 0$ and atleast one of Δ_1, Δ_2, Δ_3 is non-zero, then the solutions are	(r)	Non-Trivial
(D)	If atleast one of Δ_1, Δ_2, Δ_3 is non-zero, and $\Delta = 0$, then the solutions are	(s)	Infinitely many

PAPER-2

SECTION-I

Integer Answer Type

This section contains eight questions. The answer to each question is a single digit integer ranging from 0 to 9, both inclusive. For each question, darken the bubble corresponding to the correct integer in the ORS. Marking scheme : +4 If the bubble corresponding to the answer is darkened, 0 In all other cases.

1. Let $f(x, y)$ be a periodic function satisfying the condition $f(x, y) = f((2x + 2y), (2y - 2x))\ \forall\ x, y \in R$. Now, define a function g by $g(x) = f(2^x, 0)$. If period of $g(x)$ is α, then value of $|\alpha - 7|$ is

2. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distances of the point P from the lines $x - y = 0$ and $x + y = 0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \leq d_1(P) + d_2(P) \leq 4$, is

3. The circle $x^2 + y^2 - 4x - 4y + 4 = 0$ is inscribed in a triangle which has two of its sides along the coordinate axes. The locus of the circumcentre of the triangle is $x + y - xy + k\sqrt{x^2 + y^2} = 0$, find k.

4. If $f(x) = x^3 + x^2 f'(1) + xf''(2) + f'''(3)$ for all $x \in R$, then find the value of $f(1)$.

5. Find $\lim\limits_{x \to \infty} \left| \dfrac{(3x^4 + 2x^2)\sin\frac{1}{x} + |x|^2 + 5}{|x|^3 + |x|^2 + |x| + 1} \right|$

6. If $y(t)$ is a solution of $(1+t)\dfrac{dy}{dt} - ty = 1$ and $y(0) = -1$, then $y(1) + \dfrac{3}{2}$ is equal to

7. Suppose that f is differentiable for all x and that $f'(x) \leq 2$ for all x. If $f(1) = 2$ and $f(4) = 8$, then $f(2)$ has the value equal to______.

8. Two whole numbers are randomly selected and multiplied. If the probability that the units place in their product is "Even" is p and the probability that the units place in their product is "odd" is q, then $\left(\dfrac{p}{q}\right) + 1$ is equal to

SECTION-II

One or More Than One Option Correct Type

This section contains eight questions. Each question has four options (a), (b), (c) and (d). One or More than one of these four option (s) is (are) correct. For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS. Marking scheme : +4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened, 0 If none of the bubbles is darkened, −2 In all other cases.

9. The point of intersection of the plane $\vec{r}.(3\hat{i} - 5\hat{j} + 2k) = 6$ with the straight line passing through the origin and perpendicular to the plane $2x - y - z = 4$ is

(a) $(1, -1, -1)$ (b) $(-1, -1, 2)$

(c) $(4, 2, 2)$ (d) $\left(\dfrac{4}{3}, \dfrac{-2}{3}, \dfrac{-2}{3}\right)$

10. If the median through A of a ΔABC having vertices $A \equiv (2, 3, 5)$, $B \equiv (-1, 3, 2)$ and $C \equiv (\lambda, 5, \mu)$ is equally inclined to the axes, then

(a) $\lambda = 7$ (b) $\mu = 10$

(c) $\lambda = 10$ (d) $\mu = 7$

11. The integral of $\dfrac{1}{\sin^2 x + \tan^2 x}$ must be

(a) $-\dfrac{1}{2}\left[\tan x + \dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{1}{\sqrt{2}}\tan x\right)\right] + c$

(b) $-\dfrac{1}{2}\left[\cot x + \dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{1}{\sqrt{2}}\tan x\right)\right] + c$

(c) $-\frac{1}{2}\left[\cot x-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\tan x\right)\right]+c$

(d) $-\frac{1}{2}\left[\cot x-\frac{1}{\sqrt{2}}\cot^{-1}\left(\frac{1}{\sqrt{2}}\tan x\right)\right]+c$

12. $I=\int\sqrt{\frac{\cos^3 x}{\sin^{11} x}}\,dx$ is equal to

(a) $-\frac{2}{5}\cot^{3/2}x-\frac{2}{9}\cot^{9/2}x+c$

(b) $-\frac{2}{5}\cot^{5/2}x-\frac{2}{9}\cot^{9/2}x+c$

(c) $-\frac{2}{5}\cot^{5/2}x-\frac{2}{9}\cot^{3/2}x+c$

(d) None of these

13. The line $y = x + 5$ touches
(a) the parabola $y^2 = 20x$
(b) the ellipse $9x^2 + 16y^2 = 144$
(c) the hyperbola $\frac{x^2}{29}-\frac{y^2}{4}=1$
(d) the circle $x^2 + y^2 = 25$

14. How many numbers between 5,000 and 10,000 can be formed using the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 each digit appearing not more than once in each number ?
(a) 1680 (b) $5 \times {}^8P_8$
(c) $30 \times {}^8C_3$ (d) $5! \times {}^8C_3$

15. Triplets (p, q, r) is chosen from a set {1, 2, 3, 4, 5, 6, 7, 8, 9} such that $x < y \le z$. There could be …… such triplets.
(a) 143 (b) 200 (c) 180 (d) 120

16. If in triangle ABC, CD is the angle bisector of the angle ACB, then CD is equal to

(a) $\frac{a+b}{2ab}\cos\frac{C}{2}$ (b) $\frac{a+b}{ab}\cos\frac{C}{2}$

(c) $\frac{2ab}{a+b}\cos\frac{C}{2}$ (d) $\frac{b\sin A}{\sin\left(B+\frac{C}{2}\right)}$

SECTION-III

Comprehension Type

This section contains 2 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each of these questions has 4 choices (a), (b), (c) and (d), out of which only one is correct. Correct answer 3 marks and wrong answer –1.

Paragraph for Question. No. 17 and 18

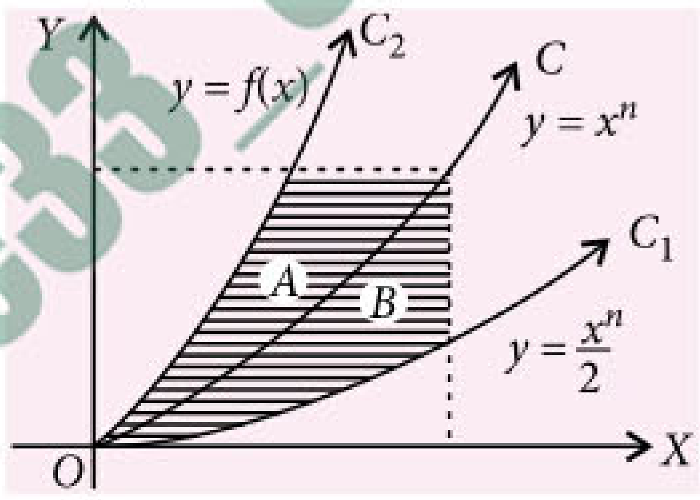
Consider a hyperbola whose centre is at origin. A line $x + y = 2$ touches this hyperbola at $P(1, 1)$ and intersect the asymptotes at A and B such that $AB = 6\sqrt{2}$ units.

17. Equation of asymptotes are
(a) $2x^2 + 5xy + 2y^2 = 0$ (b) $2x^2 - 5xy - 2y^2 = 0$
(c) $2x^2 - 5xy + 2y^2 = 0$ (d) $2x^2 + 5xy - 2y^2 = 0$

18. Equation of tangent to the hyperbola at $\left(-1,\frac{7}{2}\right)$ is
(a) $3x + 2y = 2$ (b) $3x + 4y = 11$
(c) $3x + 2y = 4$ (d) $3x + 2y = 7$

Paragraph for Question. No. 19 and 20

Let C_1 and C_2 be two curves passing through the origin as shown in the figure. A curve C is said to "bisect the area" between C_1 and C_2, if for each point P of C, the two shaded regions A and B shown in the figure have equal areas. Let the equation of the curve C_1 be $y=\frac{x^n}{2}$ and that of C be $y = x^n$.



19. Area of the total shaded region $(A + B)$, if x– coordinate of P is x and $n = 2$ is

(a) $\frac{x^2}{2}$ (b) $\frac{x^3}{2}$ (c) $\frac{x^3}{3}$ (d) $\frac{x^3}{6}$

20. Equation of the curve C_2 for $n = 2$ is $y =$

(a) $\frac{3x^3}{4}$ (b) $\frac{4x^2}{3}$ (c) $\frac{9x^2}{16}$ (d) $\frac{16x^2}{9}$

ANSWER KEY

Paper-1

1. (4) 2. (9) 3. (6) 4. (1)
5. (0) 6. (1) 7. (2) 8. (2)
9. (b, c) 10. (c) 11. (a, d) 12. (a)
13. (a, d) 14. (a, d) 15. (a, c) 16. (d)
17. (a, d) 18. (c)
19. (A) → (r), (B) → (s), (C) → (p), (D) → (q)
20. (A) → (p), (B) → (r, s), (C) → (p, r), (D) → (q)

Paper-2

1. (5) 2. (6) 3. (1) 4. (4)
5. (2) 6. (1) 7. (4) 8. (4)
9. (d) 10. (a, b) 11. (b, d) 12. (b)
13. (a, b, c) 14. (a, c) 15. (d) 16. (c, d)
17. (a) 18. (c) 19. (c) 20. (d)

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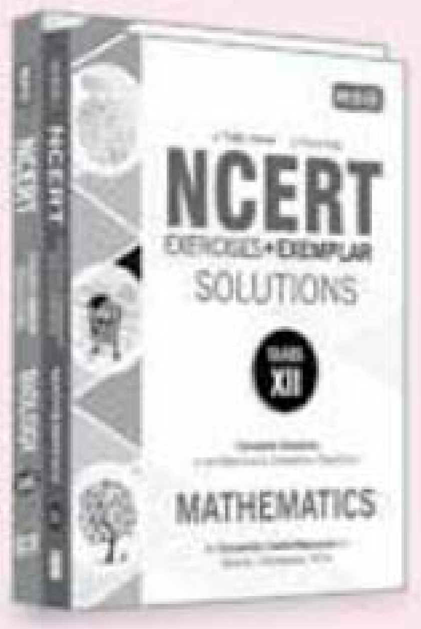
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Self check table given at the end will help you to check your readiness.

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. The largest interval lying in $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ for which the function, $f(x)=4^{-x^2}+\cos^{-1}\left(\frac{x}{2}-1\right)+\log(\cos x)$ is defined, is
   (a) $\left[-\frac{\pi}{4},\frac{\pi}{2}\right)$ (b) $\left[0,\frac{\pi}{2}\right)$
   (c) $[0, \pi]$ (d) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

2. The value of $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix}$ is
   (a) 0 (b) 1
   (c) 2 (d) none of these

3. In which of the following functions, langranges mean value theorem is NOT applicable in [0, 1]?
   (a) $f(x)=\begin{cases} \frac{1}{2}-x, & x<\frac{1}{2} \\ \left(\frac{1}{2}-x\right)^2, & x\geq\frac{1}{2} \end{cases}$
   (b) $f(x)=\begin{cases} \frac{\sin x}{x}, & x\neq 0 \\ 1, & x=0 \end{cases}$
   (c) $f(x) = x|x|$ (d) $f(x) = |x|$

4. The projection of the line $\frac{x+1}{-1}=\frac{y}{2}=\frac{z-1}{3}$ on the plane $x-2y+z=6$ is the line of intersection of this plane with the plane
   (a) $2x + y + 2 = 0$ (b) $3x + y - z = 2$
   (c) $2x - 3y + 8z = 3$ (d) none of these

5. The locus of all points on the curve $y^2=4a\left(x+a\sin\left(\frac{x}{a}\right)\right)$ at which the tangent is parallel to x-axis is
   (a) a straight line (b) a circle
   (c) a parabola (d) an ellipse

6. If $\frac{d^2x}{dy^2}\left(\frac{dy}{dx}\right)^3+\frac{d^2y}{dx^2}=k$, then k is equal to
   (a) 0 (b) 1
   (c) 2 (d) None of these

One or More Than One Option(s) Correct Type

7. Let $f: R \to R$ be a function defined by $f(x)=\frac{e^{|x|}-e^{-x}}{e^x+e^{-x}}$, then $f(x)$ is
   (a) not one-one (b) not onto
   (c) onto but not one-one
   (d) one-one but not onto

8. The probabilities that a student passes in Mathematics, Physics and Chemistry are m, p and c respectively. Of these subjects, a student has a 75% chance of passing in atleast one, 50% chance of passing in atleast two and 40% chance of passing in exactly two subjects, then which of the following relation(s) is (are) true?
   (a) $p+m+c=\frac{19}{20}$ (b) $p+m+c=\frac{27}{20}$
   (c) $pmc=\frac{1}{10}$ (d) $pmc=\frac{1}{4}$

9. If P, Q, R are angles of triangle PQR, then the value of $\begin{vmatrix} -1 & \cos R & \cos Q \\ \cos R & -1 & \cos P \\ \cos Q & \cos P & -1 \end{vmatrix}$ is equal to

(a) -1 (b) 0 (c) $\frac{1}{2}$ (d) 1

10. If $\int \frac{(2x-1)dx}{x^4-2x^3+x+1} = A\tan^{-1}\left[\frac{f(x)}{\sqrt{3}}\right] + C$, then

(a) $A=\frac{2}{\sqrt{3}}$ (b) $A = 2$

(c) $f(x) = 2x^2 - 2x - 1$ (d) $f(x) = 2x^2 + 2x + 1$

11. Non-zero vectors $\vec{a}$ and $\vec{b}$ are equally inclined to vector $\vec{c}$, if $\vec{c}$ is equal to

(a) $\frac{|\vec{b}|}{|\vec{a}|+|\vec{b}|}\vec{a}+\frac{|\vec{a}|}{|\vec{a}|+|\vec{b}|}\vec{b}$

(b) $\frac{|\vec{a}|}{|\vec{a}|+|\vec{b}|}\vec{a}+\frac{|\vec{b}|}{|\vec{a}|+|\vec{b}|}\vec{b}$

(c) $\frac{|\vec{a}|}{|\vec{a}|+2|\vec{b}|}\vec{a}+\frac{|\vec{b}|}{|\vec{a}|+2|\vec{b}|}\vec{b}$

(d) $\frac{|\vec{b}|}{2|\vec{a}|+|\vec{b}|}\vec{a}+\frac{|\vec{a}|}{2|\vec{a}|+|\vec{b}|}\vec{b}$

12. Let $f(x)=\begin{cases}\int_0^x (1+|1-t|)dt, & x>2\\ 5x+1, & x\le 2\end{cases}$, then

(a) $f(x)$ is not continuous at $x = 2$

(b) $f(x)$ is continuous but not differentiable at $x = 2$

(c) $f(x)$ is differentiable everywhere

(d) the right derivative of $f(x)$ at $x = 2$ does not exist.

13. If $D^* f(x)=\lim_{h\to 0}\frac{f^2(x+h)-f^2(x)}{h}$ where $f^2(x)=\{f(x)\}^2$

The value of $D^* f(x)$ at the point on the curve $y = f(x)$ such that tangent to it are parallel to x-axis, then

(a) $f(x)$ (b) 0 (c) $2f(x)$ (d) $xf(x)$

Comprehension Type

Let two planes $P_1 : 2x - y + z = 2$ and $P_2 : x + 2y - z = 3$ are given.

14. The equation of the acute angle bisector of planes P_1 and P_2 is

(a) $x - 3y + 2z + 1 = 0$ (b) $3x + y - 5 = 0$

(c) $x + 3y - 2z + 1 = 0$ (d) $3x + z + 7 = 0$

15. The Image of plane P_1 in the plane mirror P_2 is

(a) $x + 7y - 4x + 5 = 0$ (b) $3x + 4y - 5z + 9 = 0$

(c) $7x - y + 2z - 9 = 0$ (d) None of the above

Matrix Match Type

16. Match the following.

	Column I		Column II
P.	Let $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$, then	1.	$f(x)$ increases, if $x \in (2, 3)$
Q.	Let $f(x) = 2\ \text{In}(x-2) - x^2 + 4x + 1$, then	2.	$f(x)$ increases, if $x \in (0, 1)\cup(2, \infty)$
R.	Let $f(x)=\frac{\|x-1\|}{x^2}$, then	3.	$f(x)$ increases, if $x \in (1, 2)\cup(3, \infty)$
		4.	$f(x)$ decreases, if $x \in (0, 1)\cup(2, \infty)$

	P	Q	R
(a)	2	1	3
(b)	1	3	4
(c)	3	1	4
(d)	3	4	1

Integer Answer Type

17. If $f : R \to R$, $g : R \to R$ and $h : R \to R$ is such that $f(x) = x^2$, $g(x) = \tan x$ and $h(x) = \log x$, then the value of $[ho(gof)]\,(x)$, if $x=\frac{\sqrt{\pi}}{2}$ will be

18. The vectors $x\hat{i}-3\hat{j}+7\hat{k}$ and $\hat{i}+y\hat{j}-z\hat{k}$ are collinear, then the value of $\frac{-7xy^2}{z}$ is equal to

19. The range of the function $f(x)=\sqrt{(x-6)}+\sqrt{(12-x)}$ is an interval of length $\sqrt{\lambda}-\sqrt{\mu}$, then $3\lambda - 5\mu$ must be

20. If $S=\sum\tan^{-1}\left(\frac{2r}{2+r^2+r^4}\right)$, then one's place in the value of 2550 cot S is

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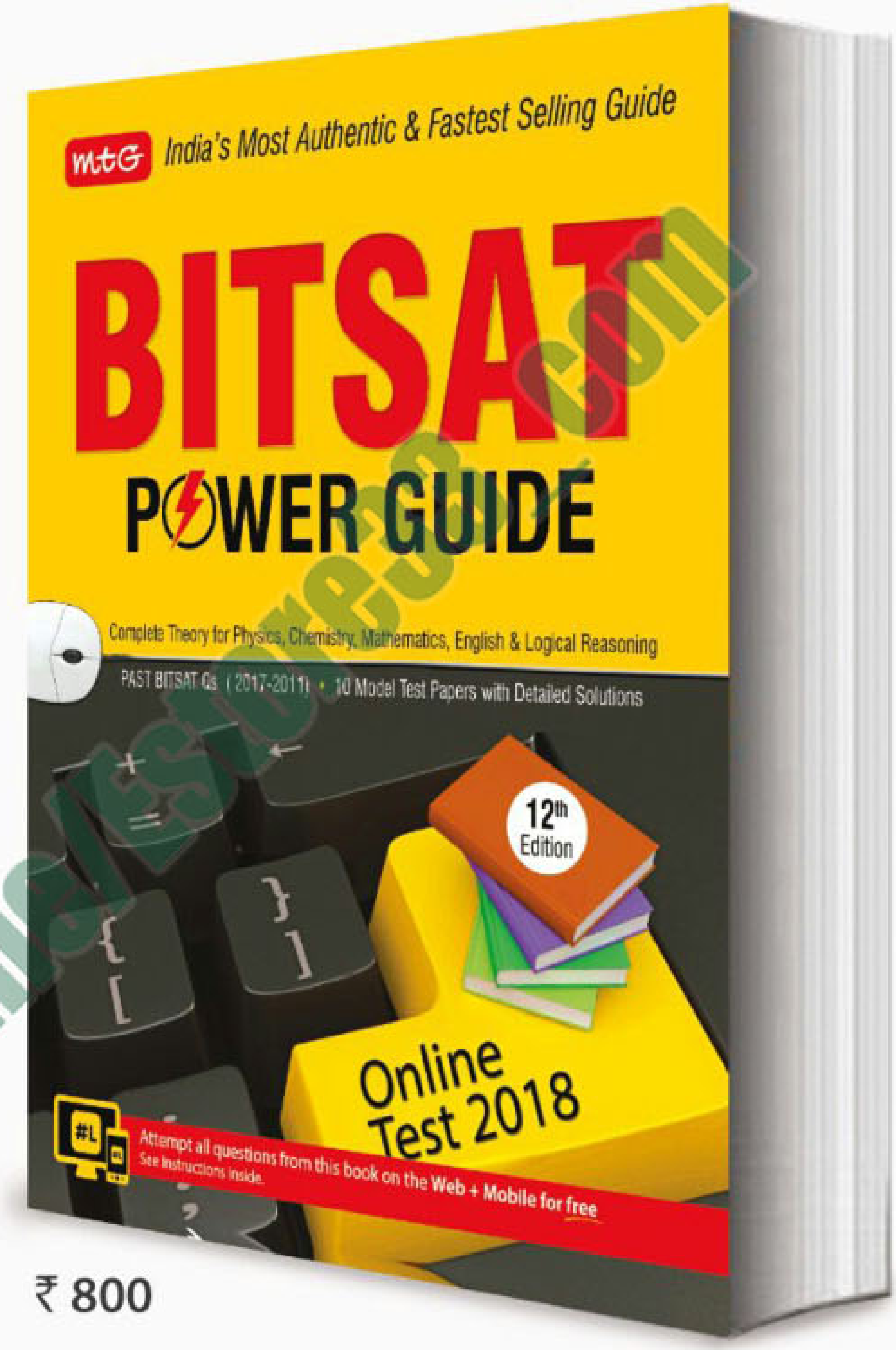
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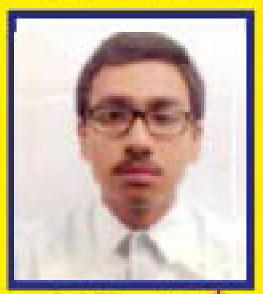
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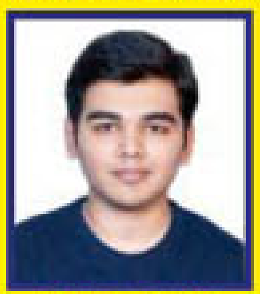
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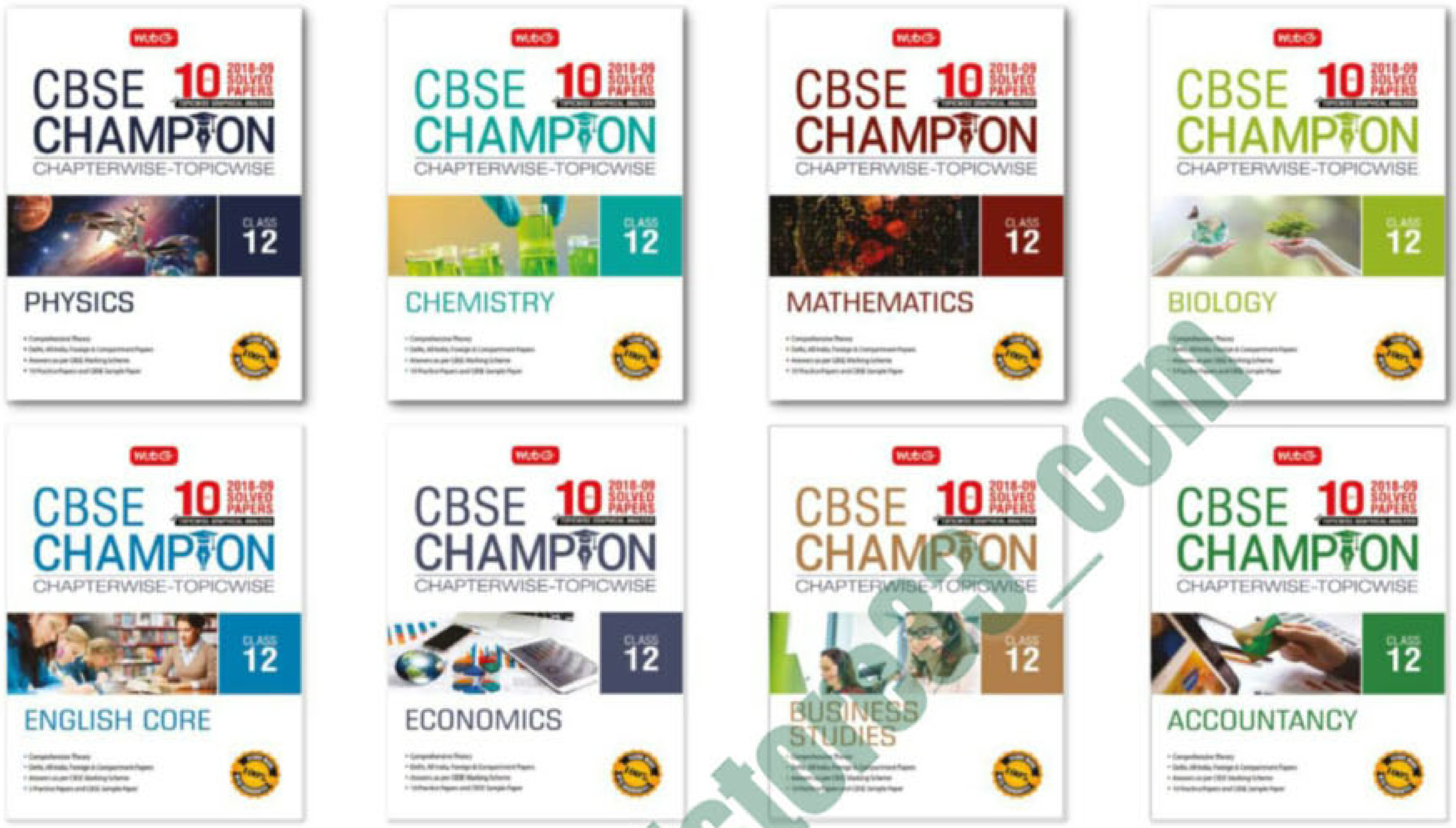
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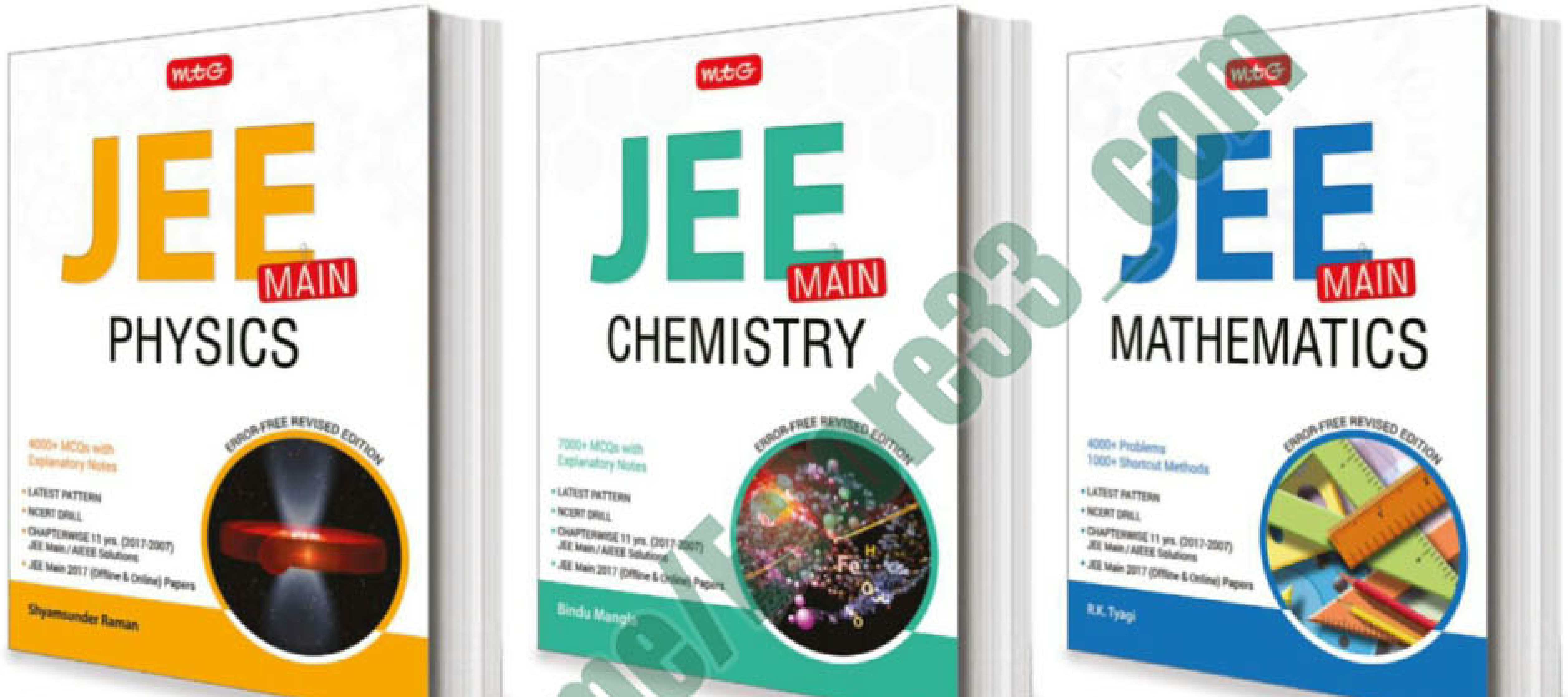
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